Mixed Integer Programming
The State of the Art

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Optimization

A Definition

A *mixed-integer program* (MIP) is an optimization problem of the form

$$
\begin{aligned}
\textit{Minimize} \quad & c^T x \\
\textit{Subject to} \quad & Ax = b \\
& l \leq x \leq u \\
\text{some or all } & x_j \text{ integer}
\end{aligned}
$$

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Unit-Commitment Models

Electrical Power Industry, ERPI GS-6401, June 1989:
Mixed-integer programming (MIP) is a powerful modeling tool, "They are, however, theoretically complicated and computationally cumbersome"

In Other Words: MIP is an interesting modeling "toy", but it just doesn't work in practice.

This perception began to change in 1999.

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From the Rutger's DIMACS Meeting 1999:
California 7-Day Model

`UNITCAL_7:` 48939 constraints, 25755 variables (2856 binary)

Reported Results 1999 – machine unknown

2 Day model: 8 hours, no progress

7 Day model: 1 hour to solve initial LP

Desktop PC -- ran full 7-day model

CPLEX 6.5 (1999): 22 minutes, optimal

TODAY (2015): 15 seconds, optimal

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Computational History: 1950 -1998

- 1954 Dantzig, Fulkerson, S. Johnson: 42 city TSP
  - Solved to optimality using LP and cutting planes
- 1957 Gomory
  - Cutting plane algorithms
- 1960 Land, Doig; 1965 Dakin
  - B&B
- 1964-68 LP/90/94
  - First commercial application
- IBM 360 computer
  - 1974 MPSX/370
  - 1976 Sciconic
    - LP-based B&B
    - MIP became commercially viable

- 1975 – 1998 Good B&B remained the state-of-the-art in commercial codes, in spite of ….
  - Edmonds, polyhedral combinatorics
  - 1973 Padberg, cutting planes
  - 1973 Chvátal, revisited Gomory
  - 1974 Balas, disjunctive programming
  - 1983 Crowder, Johnson, Padberg: PIPX, pure 0/1 MIP
  - 1987 Van Roy and Wolsey: MPSARX, mixed 0/1 MIP
  - TSP, Grötschel, Padberg, …

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1998 … A New Generation of MIP Codes

- Linear programming
  - Stable, robust dual simplex
- Variable/node selection
  - Influenced by traveling salesman problem
- Primal heuristics
  - 12 different tried at root
  - Retried based upon success
- Node presolve
  - Fast, incremental bound strengthening (very similar to Constraint Programming)

- Presolve – numerous small ideas
  - Probing in constraints:

    $\sum x_j \leq (\sum u_j)\, y, \quad y = 0/1$

    ➔ $x_j \leq u_j y$ (for all j)
- Cutting planes
  - Gomory, mixed-integer rounding (MIR), knapsack covers, flow covers, cliques, GUB covers, implied bounds, zero-half cuts, path cuts

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MIP Speedups

Some Test Results

- Test set: **1852 real-world MIPs**
  - Full library
    - 2791 MIPs
  - Removed:
    - 559 "Easy" MIPs
    - 348 "Duplicates"
    - 22 "Hard" LPs (0.8%)
- Parameter settings
  - Pure defaults
  - 30000 second time limit
- Versions Run
  - CPLEX 1.2 (1991) -- CPLEX 11.0 (2007)

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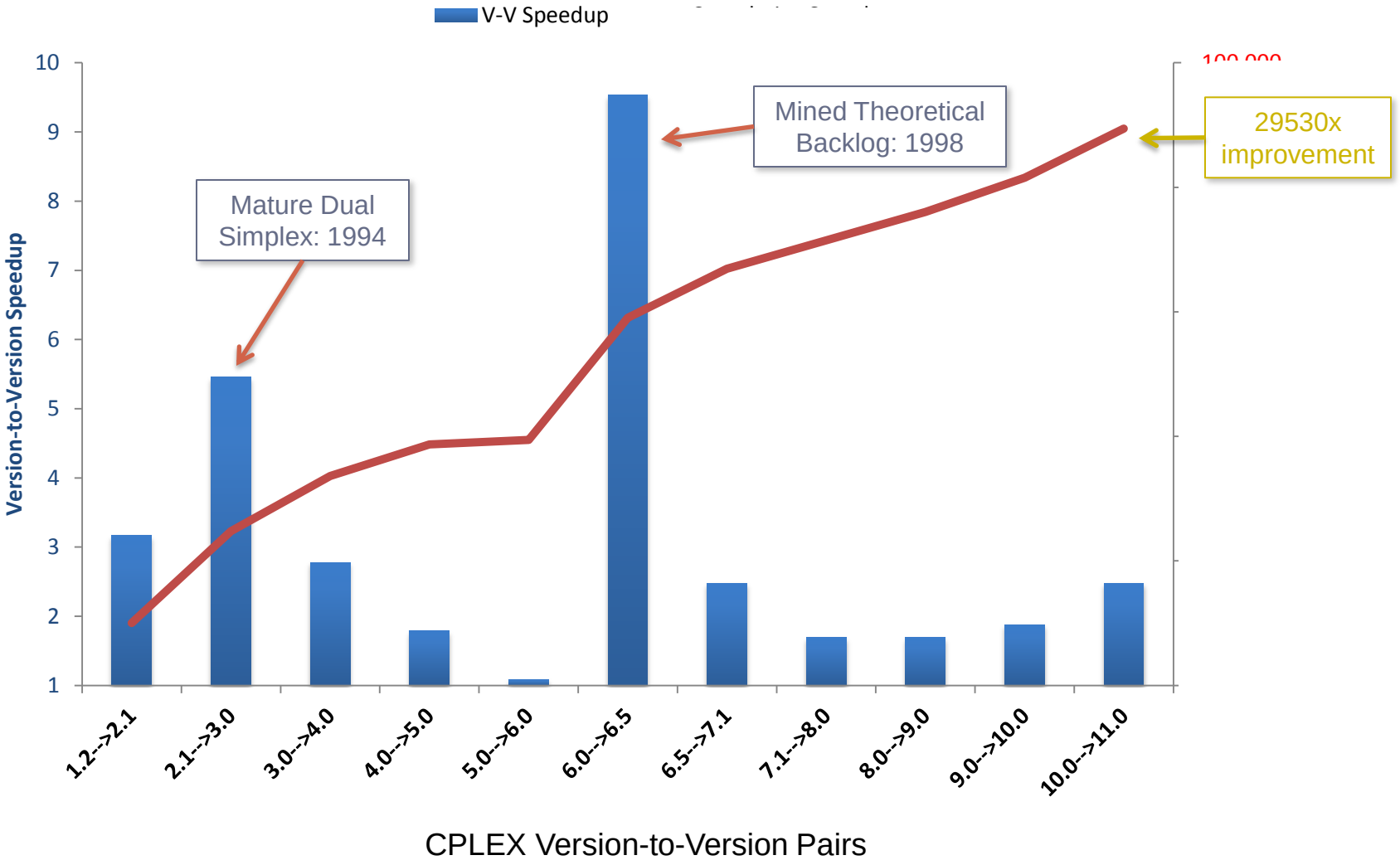
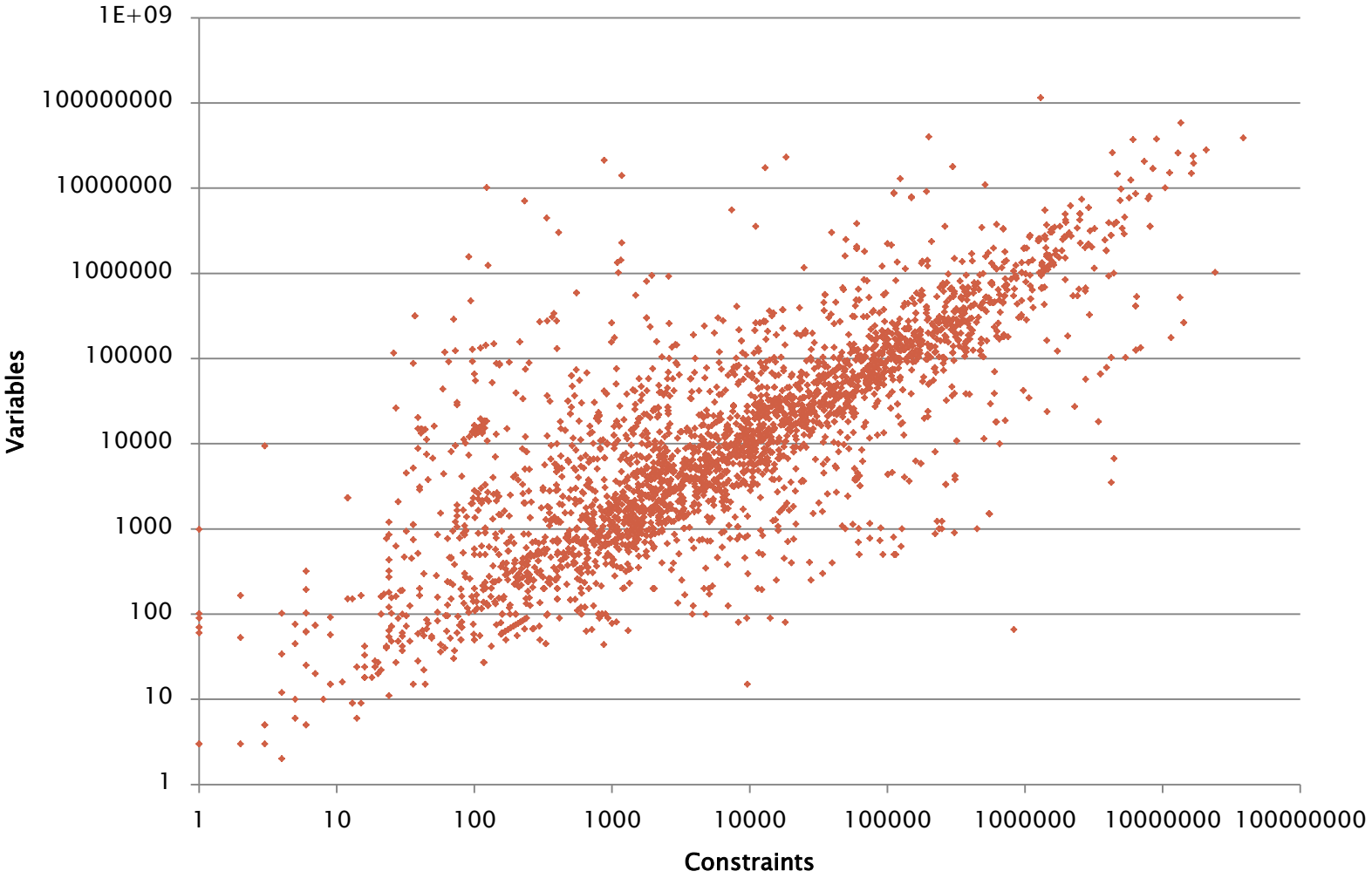
CPLEX Version Performance Improvements (1991–2008)



Progress: 2009 – Present

Gurobi MIP Library

(3550 models)



MIP Speedup 2009-Present

- Starting point
  - Gurobi 1.0 & CPLEX 11.0 ~equivalent on 4-core machine

- Gurobi Version-to-version improvements
  - Gurobi 1.0 -> 2.0: 2.4X
  - Gurobi 2.0 -> 3.0: 2.2X (5.1X)
  - Gurobi 3.0 -> 4.0: 1.3X (6.6X)
  - Gurobi 4.0 -> 5.0: 2.0X (12.8X)
  - Gurobi 5.0 -> 6.0: 2.2X (27.6X)
  - Gurobi 6.0 -> (6.5): 1.7X (46.0X)

- Machine-independent IMPROVEMENT since 1991
  - Over 1.3 million X -- 1.8X/year

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Suppose you were given the following choices:

- **Option 1**: Solve a MIP with today's solution technology on a machine from 1991
- **Option 2**: Solve a MIP with 1991 solution technology on a machine from today

Which option should you choose?

- **Answer**: **Option 1 would be faster by a factor of approximately 300.**

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Thank you