# ILLUSTRATING TWO MODELS OF LOCATIONAL OPERATING RESERVE DEMAND CURVES

**William W. Hogan**

***Mossavar-Rahmani Center for Business and Government***
***John F. Kennedy School of Government***
***Harvard University***
***Cambridge, Massachusetts  02138***

**Harvard Electricity Policy Group**

**Cambridge, MA**

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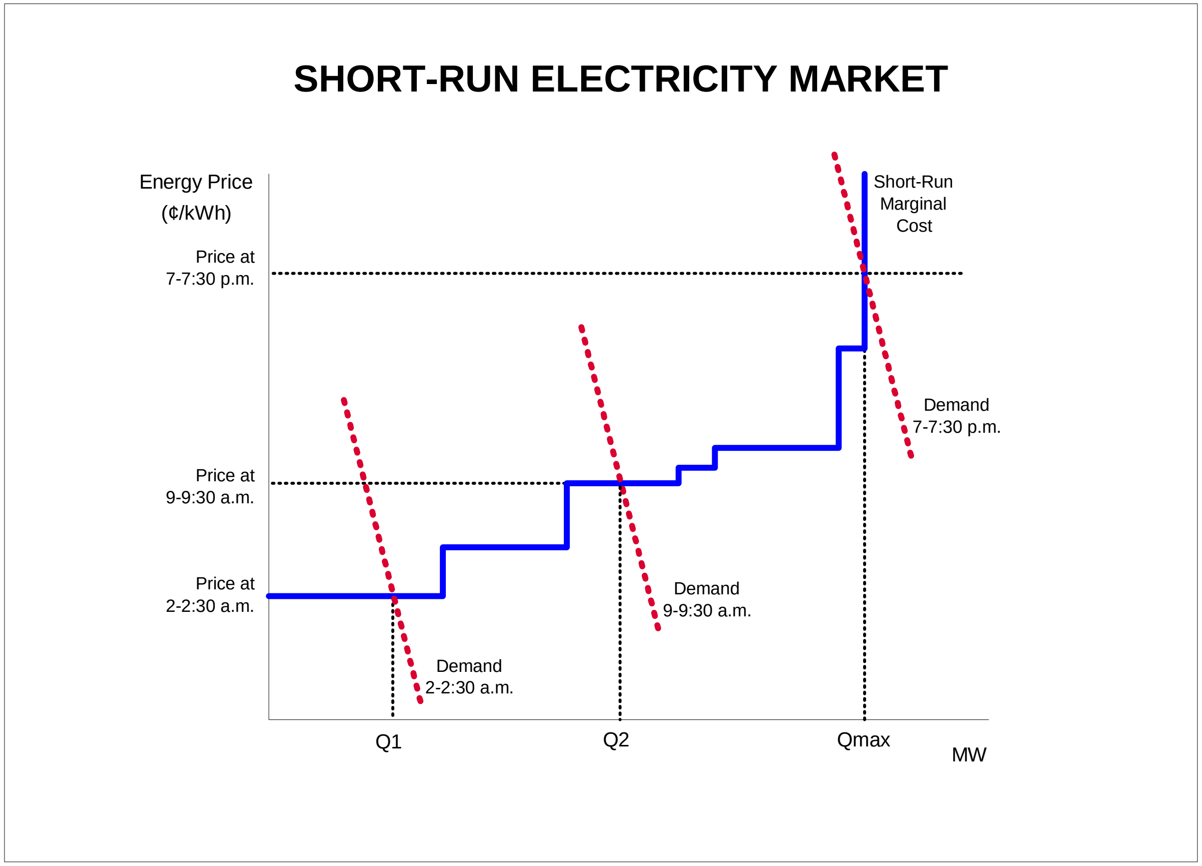
**Scarcity pricing presents an important challenge for Regional Transmission Organizations (RTOs) and electricity market design. Simple in principle, but more complicated in practice, inadequate scarcity pricing is implicated in several problems associated with electricity markets.**

- **Investment Incentives.** Inadequate scarcity pricing contributes to the "missing money" needed to support new generation investment. The policy response has been to create capacity markets. Better scarcity pricing would reduce the challenges of operating good capacity markets.
- **Demand Response.** Higher prices during critical periods would facilitate demand response and distributed generation when it is most needed. The practice of socializing payments for capacity investments compromises the incentives for demand response and distributed generation.
- **Renewable Energy.** Intermittent energy sources such as solar and wind present complications in providing a level playing field in pricing. Better scarcity pricing would reduce the size and importance of capacity payments and improve incentives for renewable energy.
- **Transmission Pricing.** Scarcity pricing interacts with transmission congestion. Better scarcity pricing would provide better signals for transmission investment.

**Smarter scarcity pricing would mitigate or substantially remove the problems in all these areas. While long-recognized, the need for smarter prices for a smarter grid promotes interest in better theory and practice of scarcity pricing.**[1]

[1] FERC, Order 719, October 17, 2008.

**Early market designs presumed a significant demand response. Absent this demand participation most markets implemented inadequate pricing rules equating prices to marginal costs even when capacity is constrained. This produces a "missing money" problem.**



**The theory and practice of scarcity pricing intersect important elements of electricity systems and economic dispatch.**

- **Reliability.** By definition, scarcity conditions arise when the system is constrained and dispatch is modified to respect reliability constraints.
- **Dispatch.** Simultaneous optimization of energy and reserves means that scarcity in either effects prices for both.
- **Resource Adequacy.** The standards for resource adequacy and operating security are not fully integrated or compatible.

**A critical connection is the treatment of operating reserves and construction of operating reserve demand curves. The basic idea of applying operating reserve demand curves is well tested and found in operation in important RTOs.**

- **NYISO.** See NYISO Ancillary Service Manual, Volume 3.11, Draft, April 14, 2008, pp, 6-19-6-22.
- **ISONE.** FERC Electric Tariff No. 3, Market Rule I, Section III.2.7, February 6, 2006.
- **MISO.** FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009. [2]

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[2] "For each cleared Operating Reserve level less than the Market-Wide Operating Reserve Requirement, the Market-Wide Operating Reserve Demand Curve price shall be equal to the product of (i) the Value of Lost Load ("VOLL") and (ii) the estimated conditional probability of a loss of load given that a single forced Resource outage of 100 MW or greater will occur at the cleared Market-Wide Operating Reserve level for which the price is being determined. … The VOLL shall be equal to $3,500 per MWh." MISO, FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009, Sheet 2226.

**A difficulty with defining a locational operating reserve demand curve is the complexity of the interactions among locations plus interactions with the transmission grid. A similar problem appears in the evaluation of planned transmission and generation investment.**

- **Expected Values.** The basic formulation of the real-time economic dispatch problem is built on a particular configuration of the transmission grid and the usual application of Kirchoff's laws. The operating reserve and long-term planning problem share a focus on the expected values of outcomes across different conditions. The expected value in principle applies probabilities across many configurations and the expected value need not follow the individual dictates of Kirchoff's laws.

- **Zonal Model.** The expected value formulation rationalizes approximation in a zonal model. The zonal application across a wide range of conditions is a regular feature of RTO transmission planning and resource adequacy calculations.

  - **Zones with Closed Interfaces.** Areas with limited transmission are defined and treated as having a close interface with a capacity limit for emergency transfers from the rest of the system.

  - **Capacity Emergency Transfer Limit (CETL).** Conservative transmission standards (e.g., 1 day in 25 years) apply to simulations that determine the transfer limit.[3]

- **Interface Limits.** Although the exact CETL calculations might not be appropriate for short-term reserve management, the analogy could be applied to determine closed interface limits.

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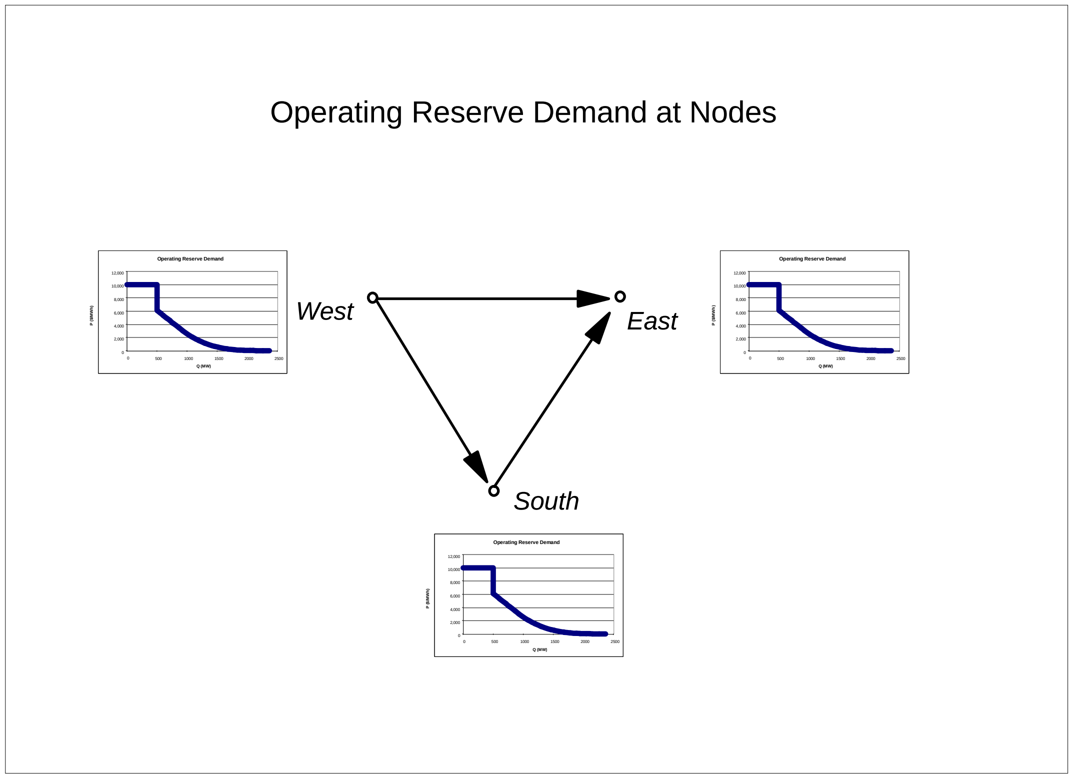
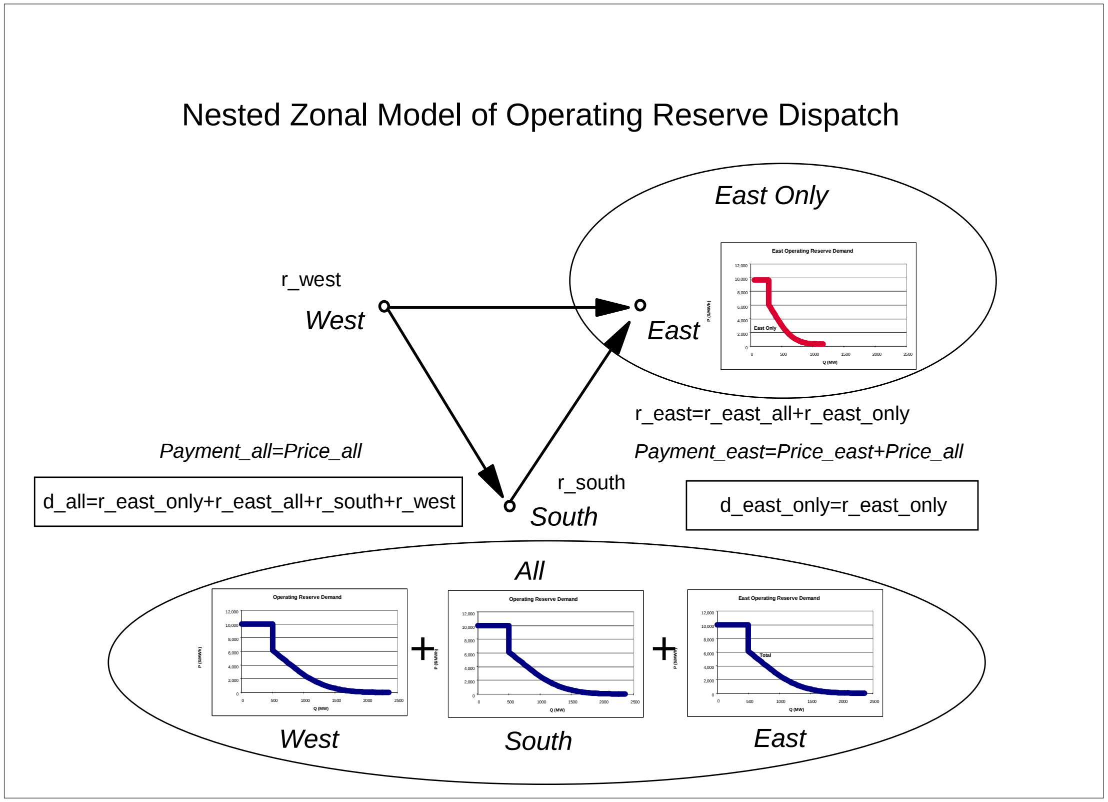
3 PJM , *2008 PJM Reserve Requirement Study*, October 8, 2008, Appendix H.

**Suppose that the *LOLP* distribution at each node could be calculated.[4] This would give rise to an operating reserve demand curve at each node.**



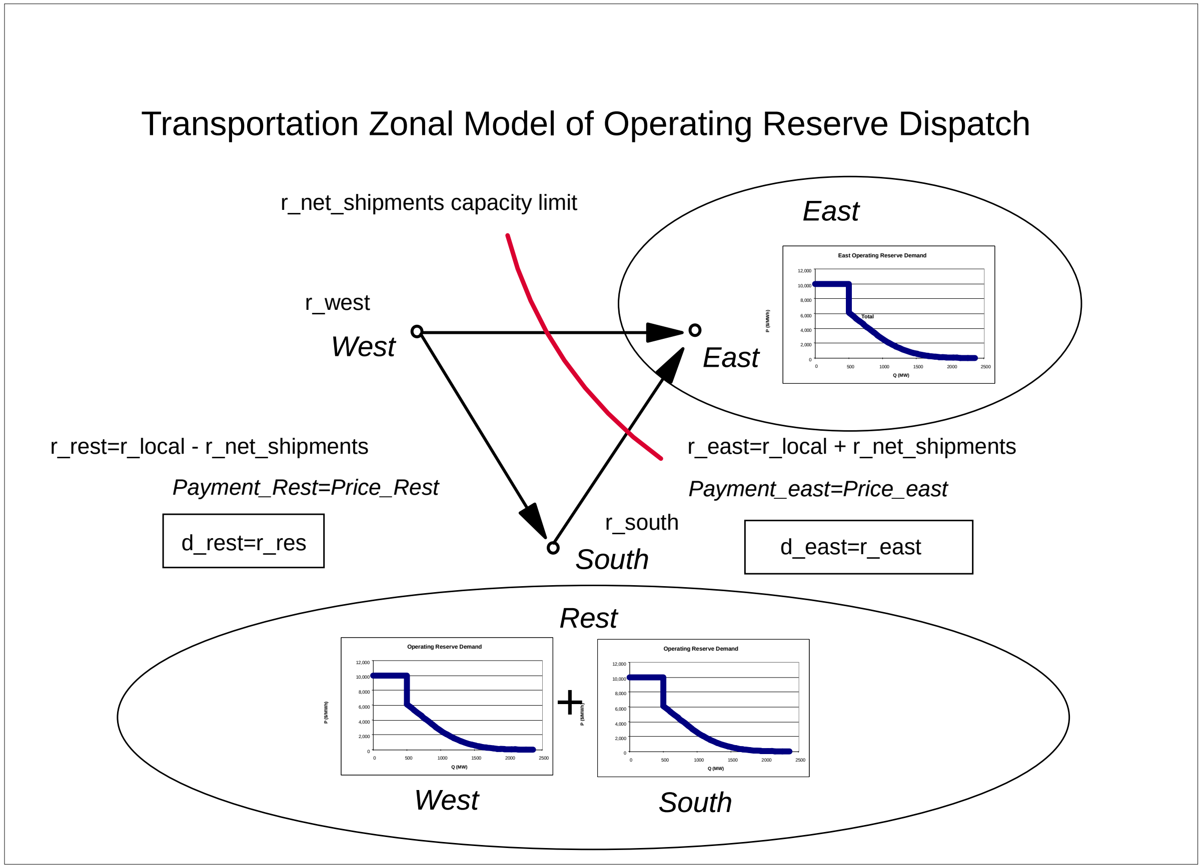

[4] Eugene G. Preston, W. Mack Grady, Martin L. Baughman, "A New Planning Model for Assessing the Effects of Transmission Capacity Constraints on the Reliability of Generation Supply for Large Nonequivalenced Electric Networks," IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1367-1373. J. Choi, R. Billinton, and M. Futuhi-Firuzabed, "Development of a Nodal Effective Load Model Considering Transmission System Element Unavailabilities," IEE Proceedings - Generation, Transmission and Distribution, Vol. 152, No. 1, January 2005, pp. 79-89.

**The next piece is a model of simultaneous dispatch of operating reserves and energy. One approach for the operating reserve piece is a cascading zonal model (e.g., NYISO reserve pricing).**



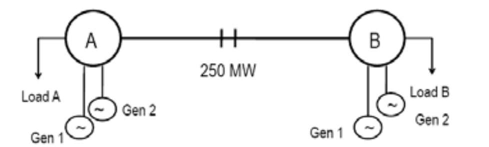

The result is that the input operating reserve price functions are additive premiums that give rise to an implicit operating reserve demand curves with higher prices.

**An alternative approach would be to overlay a transportation model with interface transfer limits on operating reserve "shipments." The resulting prices are on the demand curves, but the model requires estimation of the (dynamic) transfer capacities. This is similar to the PJM installed capacity deliverability model, but specified an hour ahead rather than years ahead.**



**A PJM example illustrates a cascading model of operating reserve demand curves of the type in use in RTOs. There is a reserve demand for Zone B, and a reserve demand for the total including Zone A and Zone B. Transmission capacity can be used for energy or reserved for operating reserves. The reserves have individual limits (e.g., ramping) and joint limits with energy.[5]**

Operating Reserve Demand Curve Example

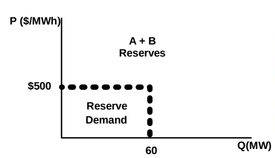


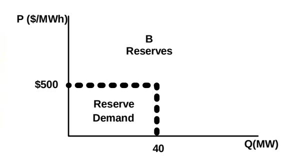

Example Information

| Generator | Energy Offer ($/MWh) | (Eco Min) MW | Total Capacity (Eco Max) (MW) | 10 minute Reserve Capability (MW) |
|---|---|---|---|---|
| Zone A | | | | |
| 1 | $20 | 0 | 600 | 60 |
| 2 | $\$20 + \frac{1}{2} P_2^A$ | 0 | 400 | 40 |
| Zone B | | | | |
| 1 | $60 | 0 | 400 | 40 |
| 2 | $\$100 + P_2^B$ | 0 | 300 | 30 |

10 Minute Reserve Requirement and Penalty Factors

| Zone | Requirement (MW) | Penalty Factor ($/MW) |
|---|---|---|
| A+B (RTO) | 60 | $500 |
| B | 40 | $500 |



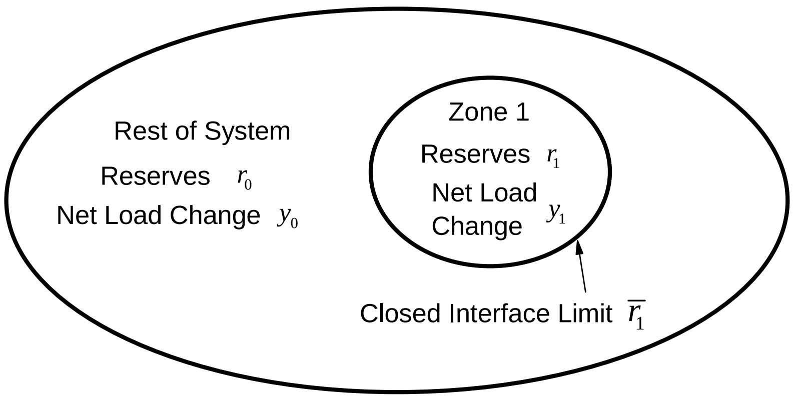

5 PJM, "ORDC/RCPF Example to Show Locational Price Impacts-Part 1," Scarcity Pricing Working Group, September 3, 2009.

**Different variants of operating reserve demand curves can be and have been integrated with energy dispatch. A challenge for any locational operating reserve demand curve is to define a framework for deriving the form of the demand curve.**

- **Generalize Loss of Load Probability (LOLP) and expected unserved energy from the aggregate system.** The simple model of loss of load from random changes in demand and generation provides a starting point but does not address locational interactions.
- **Integrate reservation of interface capacity.** A zonal model of interface capacity would include tradeoffs between normal energy dispatch and reservation of interface capacity to allow transfer of operating reserves.
- **Derive interaction between reserves in different locations.** Under some conditions, reserves in one location can support outages in another location.

**The task is to define a locational operating reserve model that approximates and prices the dispatch decisions made by operators. To illustrate, consider the simplest case with one constrained zone and the rest of the system. The reserves are defined separately and there is a known transfer limit for the closed interface between the constrained zone and the rest of the system.**

Zonal Interface Limit on Emergency Transfers



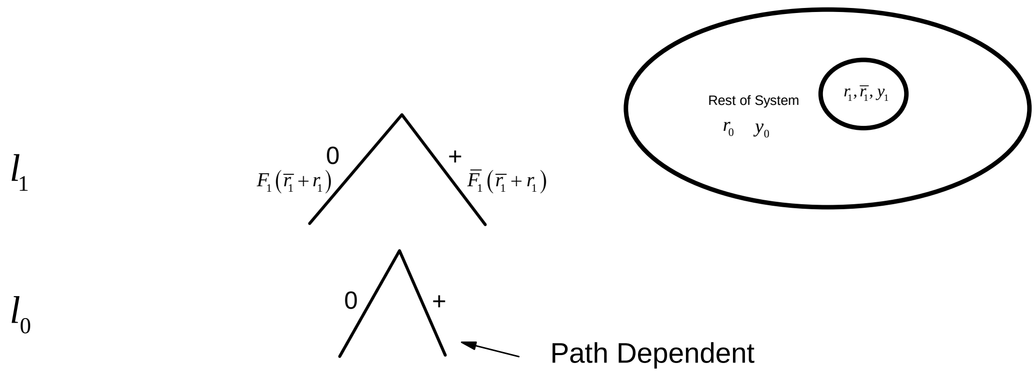

$$y_0 \sim f_0, y_1 \sim f_1, F_o(y_0) = \int_{-\infty}^{y_0} f_0(x_0)\,dx_0, F_1(y_1) = \int_{-\infty}^{y_1} f_1(x_1)\,dx_1$$

**The zonal value of expected unserved energy (ZVEUE) would be an added component of the objective function in economic dispatch. The basic problem determines the configuration of lost load. The derivatives of ZVEUE define the demand curves for operating reserves.**

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$

## Loss of Load Probability Structure

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq \overline{r}_1+r_1\right\}\right]$$



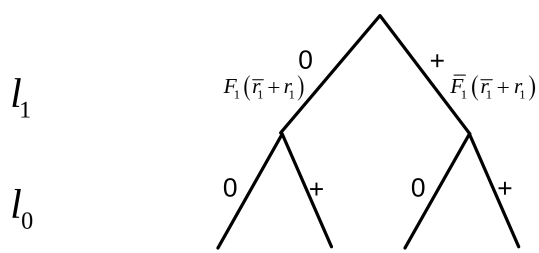

$$y_i \sim f_i,\, F_i\left(y_i\right)=\int_{-\infty}^{y_i} f_i\left(x_i\right)dx_i \qquad VOLL_0=v_0\leq VOLL_1=v_1.$$

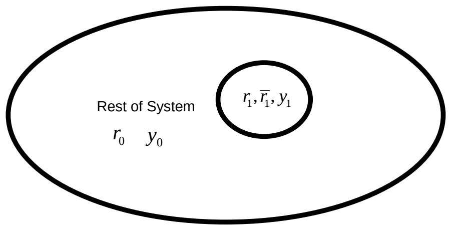
**The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves**

## Loss of Load Probabilities

$$ZVEUE\left(r_0, \overline{r}_1, r_1\right) = E_y\left[\underset{l_i \geq 0}{Min} \quad \left\{v_0 l_0 + v_1 l_1 \middle| y_0 + y_1 - l_0 - l_1 \leq r_0 + r_1, y_1 - l_1 \leq \overline{r}_1 + r_1\right\}\right]$$

Reserve Incremental Values



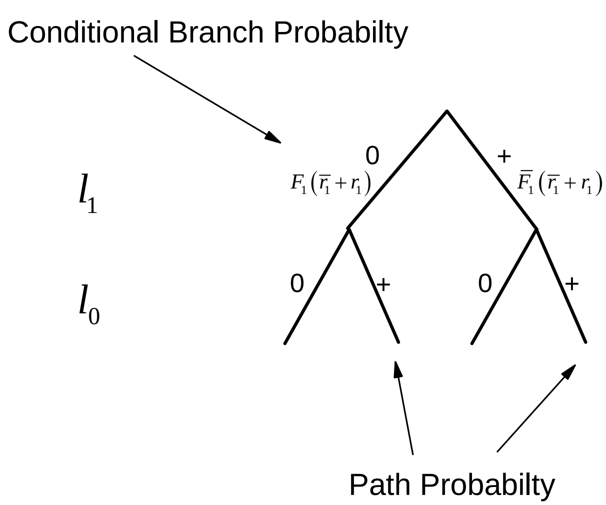



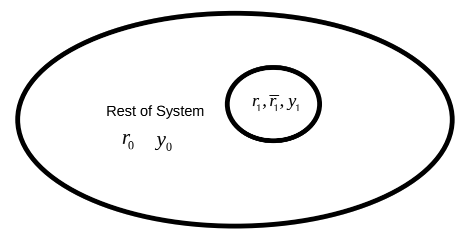

| | | | | |
|---|---|---|---|---|
| $r_1$ | 0 | $v_0$ | $v_1$ | $v_1$ |
| $\overline{r}_1$ | 0 | 0 | $v_1$ | $v_1 - v_0$ |
| $r_0$ | 0 | $v_0$ | 0 | $v_0$ |

**The full ZVEUE is difficult to characterize and calculate. However, inspection of the possible configurations of outages reveals the probabilities for the possible marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.**

## Loss of Load Probabilities

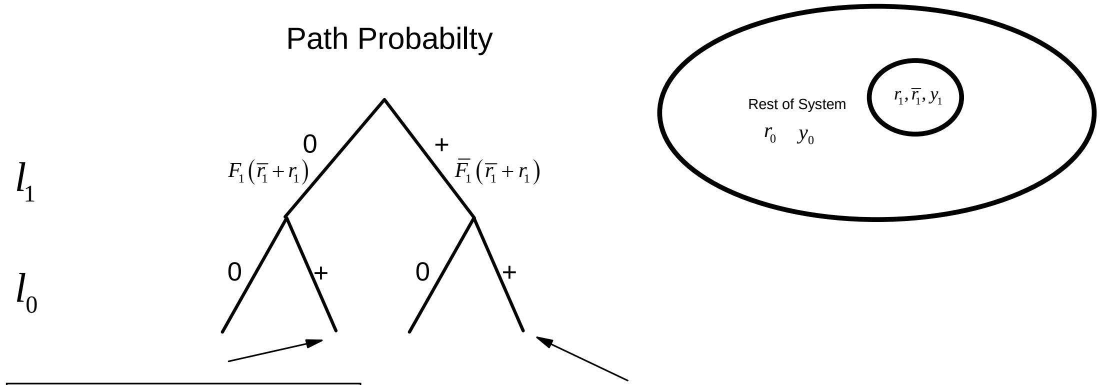
$$ZVEUE\left(r_0, \overline{r}_1, r_1\right) = E_y\left[\underset{l_i \geq 0}{Min} \quad \left\{ v_0 l_0 + v_1 l_1 \middle| y_0 + y_1 - l_0 - l_1 \leq r_0 + r_1, y_1 - l_1 \leq \overline{r}_1 + r_1 \right\}\right]$$



**Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.**

## Demand Curve Elements



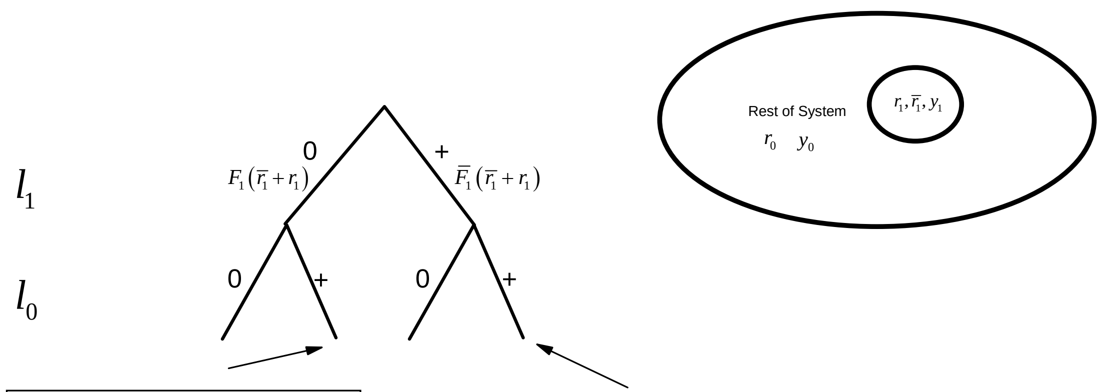

$$\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i = \int_{-\infty}^{\bar{r}_1+r_1}\bar{F}_0(r_0+r_1-x_1)\,f_1(x_1)\,dx_1$$

$$\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i = \bar{F}_1(\bar{r}_1+r_1)\,\bar{F}_0(r_0-\bar{r}_1)$$

**Assuming locational independence of outages, it is straightforward to calculate the probabilities on each path. The loss of load probabilities times the locational VOLL yields the operating reserve demand as a function of all the locational reserves and interface capacities.**

## Demand Curve Elements: Rest of System

$$p_{r_0} = v_o\left[\int_{-\infty}^{\bar{r}_1+r_1} \bar{F}_0\left(r_0+r_1-x_1\right) f_1\left(x_1\right)dx_1 + \bar{F}_1\left(\bar{r}_1+r_1\right)\bar{F}_0\left(r_0-\bar{r}_1\right)\right]$$



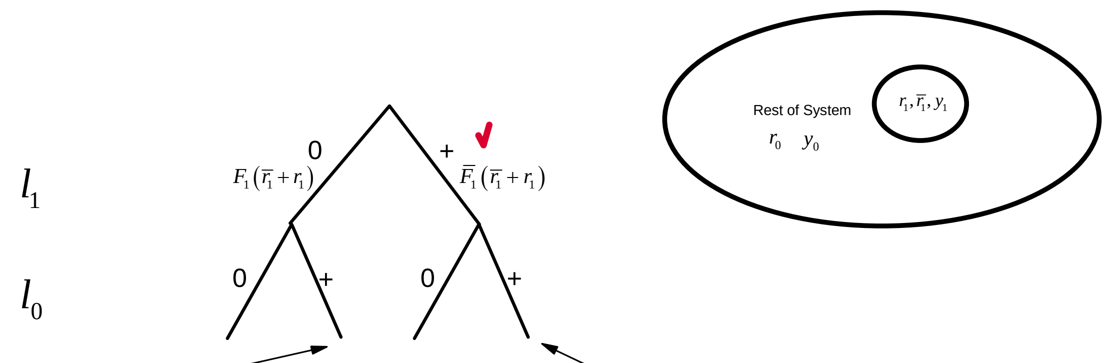

$$\int_{-\infty}^{\bar{r}_1+r_1}\int_{r_0+r_1-x_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \int_{-\infty}^{\bar{r}_1+r_1}\bar{F}_0\left(r_0+r_1-x_1\right) f_1\left(x_1\right)dx_1$$

$$\int_{\bar{r}_1+r_1}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{1} f_i\left(x_i\right)dx_i = \bar{F}_1\left(\bar{r}_1+r_1\right)\bar{F}_0\left(r_0-\bar{r}_1\right)$$

**A similar inspection of the possible paths in the trees identifies the probability that an increment of operating reserve would change the unserved energy. The possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.**

## Demand Curve Elements: Zone 1

$$p_{r_1} = v_1 \bar{F}_1\left(\bar{r}_1 + r_1\right) + v_o \left[ \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1 \right]$$



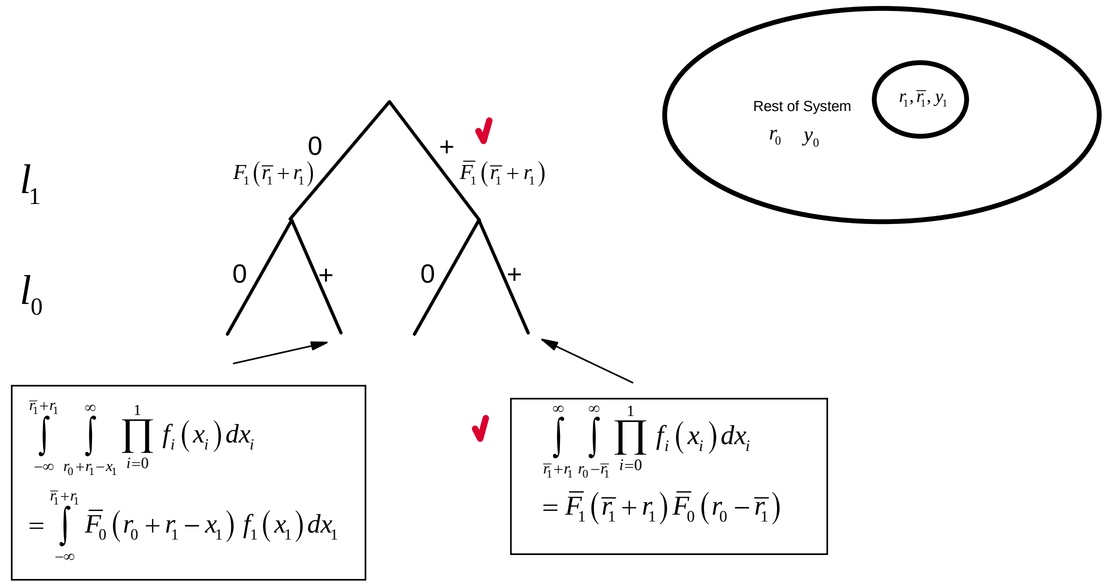

$$\int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \int_{-\infty}^{\bar{r}_1 + r_1} \bar{F}_0\left(r_0 + r_1 - x_1\right) f_1\left(x_1\right) dx_1$$

$$\int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i = \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_0\left(r_0 - \bar{r}_1\right)$$

**A similar calculation provides the demand for interface capacity as a function of the level of locational operating reserves and interface capacity.**

## Demand Curve Elements: Interface

$$p_{\bar{r}_1} = v_1 \bar{F}_1\left(\bar{r}_1 + r_1\right) - v_0\left[\bar{F}_1\left(\bar{r}_1 + r_1\right)\bar{F}_0\left(r_0 - \bar{r}_1\right)\right]$$



**The probability trees provide a workable means for beginning with the locational probability distributions of load and outages and calculating the resulting demand curves. The appendix outlines the extensions to multiple nested and parallel zones.**

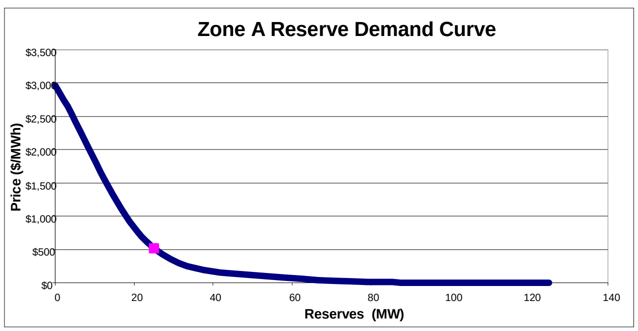
**The implied demand curves illustrate critical properties.**

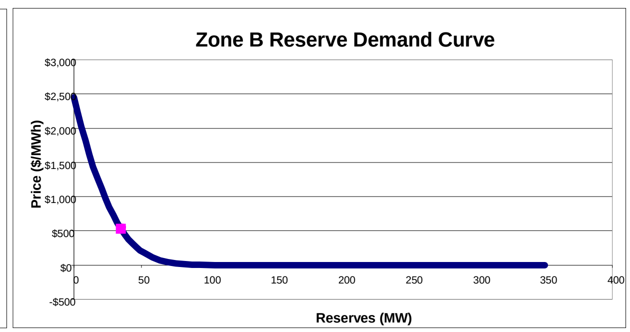
- **Interaction.** The demand curves are interdependent, but the dependence is not in the form of the nested or cascading model often assumed.
- **Maximum Value.** The value of loss load in the zone is an upper bound for the reserve price in the zone.
- **Convergence.** As the interface capacity increases, the implied demand curves in the constrained zone and for the rest of the system converge to the same prices.
- **Interface Demand.** In addition to the demand for operating reserves, there is an implied demand curve for the interface transfer limit.
- **No Thresholds.** The implied demand curve scarcity prices are positive at all levels. At higher reserves the prices are lower, but there is no threshold where the scarcity price falls to zero.

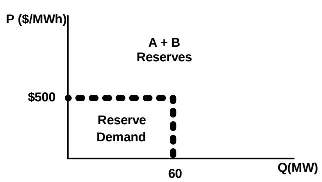
**To illustrate application of the interdependent zonal model and the cascading zonal model in the PJM example, requires the underlying outage distribution. The benchmark choice of parameters approximates the assumptions of the PJM cascade model with infinite interface capacity.**

## Benchmarking Operating Reserve Demand

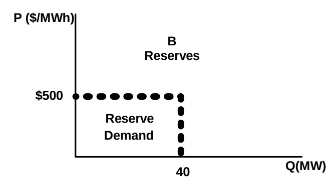
Load in: Zone A =500MW, Zone B=700MW




| Outages | Zone A | Zone B |
|---|---|---|
| Expected Total (MW) | 9.60 | 14.40 |
| Std Dev (MW) | 16.64 | 24.96 |
| VOLL ($/MWh) | 6000 | 10000 |



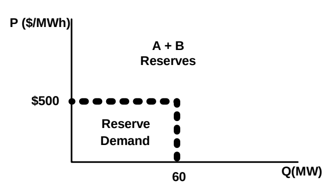
**At the benchmark load of Zone A at 500 MW and Zone B at 700 MW, economic dispatch with the cascade model produces energy and reserve dispatch with associated prices. Reserve prices are positive because of the energy redispatch required to maintain reserve levels.**

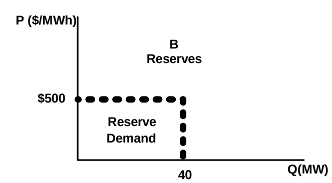
## Dispatch and Prices at Benchmark Load

pjm

No Reserve Shortage, Congestion
Load A = 500, Load B = 700 MW

| Generator | Running Cost ($/MWh) | Output (MW) | Reserve (MW) | Energy Price ($/MWh) | Reserve Price ($/MWh) |
|---|---|---|---|---|---|
| Zone A | | | | $90 | $0 |
| 1 | $20 | 600 | 0 | | |
| 2 | $\$20 + \frac{1}{2} P_2^A = \$90$ | 140 | 40 | | |
| Zone Total | | 740 | 40 | | |
| Zone B | | | | $160 | $70 |
| 1 | $60 | 400 | 0 | | |
| 2 | $\$100 + P_2^B = \$160$ | 60 | 30 | | |
| Flow A to B | | 240 | 10 | | |
| Deliverable | | 700 | 40 | | |
| RTO Total | | 1200 | 70 | | |

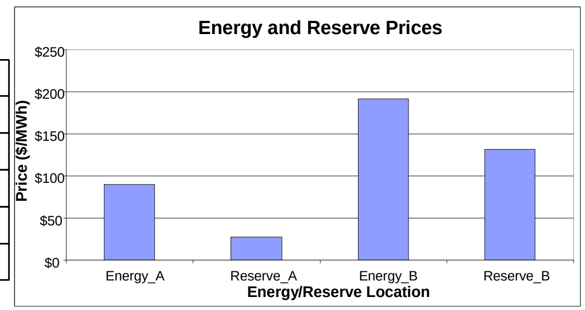


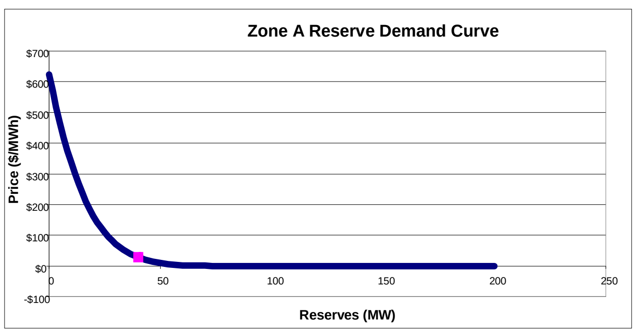



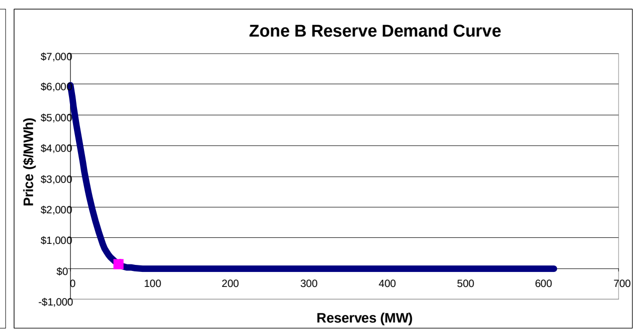
**At the benchmark loads, the interdependent locational demand curves yield similar dispatch and locational prices. However, both reserve prices are positive, reflecting the continuous nature of the alternative operating reserve demand curve. The figure indicates the local projection of the operating reserve demand curves at the economic dispatch solution.**

## Dispatch and Prices at Benchmark Load

| | | | |
|---|---|---|---|
| Load A | 500 | Load B | 700 |
| | Energy | Reserves | |
| GA_1 | 600 | 0 | |
| GA_2 | 140 | 40 | |
| GB_1 | 368 | 32 | |
| GB_2 | 92 | 30 | |



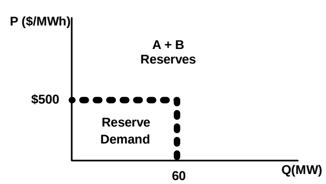



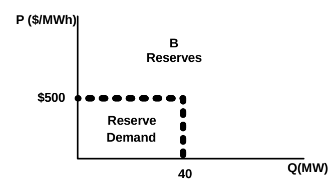
**Increasing the load in Zone B fully triggers reserve shortages and the assumed operating reserve penalty factor of $500/MWh.**

## Dispatch and Prices at High Load

Reserve Shortage in Zone B, Congestion
Load A = 500, Load B = 925 MW

| Generator | Running Cost ($/MWh) | Output (MW) | Reserve (MW) | Energy Price ($/MWh) | Reserve Price ($/MWh) |
|---|---|---|---|---|---|
| Zone A | | | | $95 | $0 |
| 1 | $20 | 600 | 0 | | |
| 2 | $\$20 + \frac{1}{2} P_2^A = \$95$ | 150 | 40 | | |
| Zone Total | | 750 | 40 | | |
| Zone B | | | | $875 | $500 |
| 1 | $60 | 400 | 0 | | |
| 2 | $\$100 + P_2^B = \$375$ | 275 | 25 | | |
| Flow A to B | | 250 | 0 | | |
| Deliverable | | 925 | 25 | | |
| RTO Total | | 1425 | 65 | | |

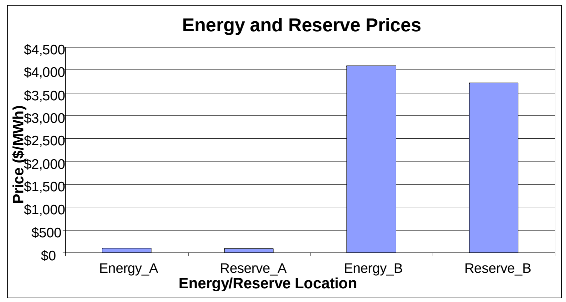


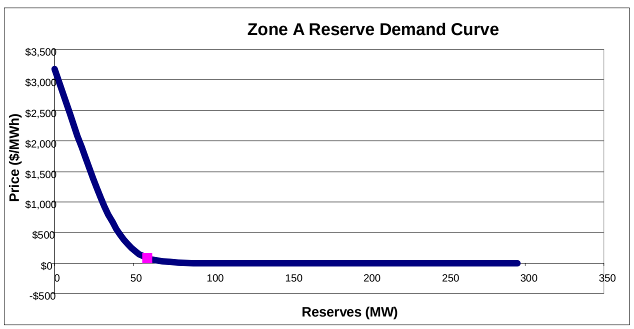



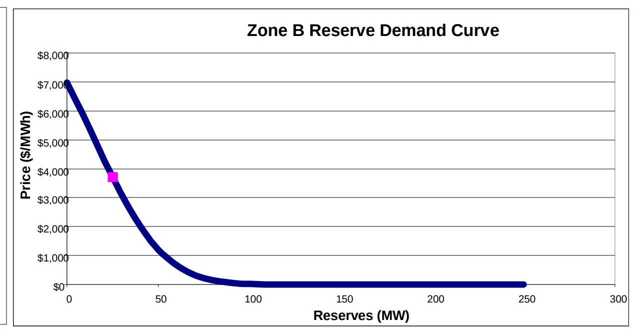
**At high load, with the implied shortage of operating reserve, the demand prices for reserves and energy increase substantially in the constrained Zone B.**

## Dispatch and Prices at High Load

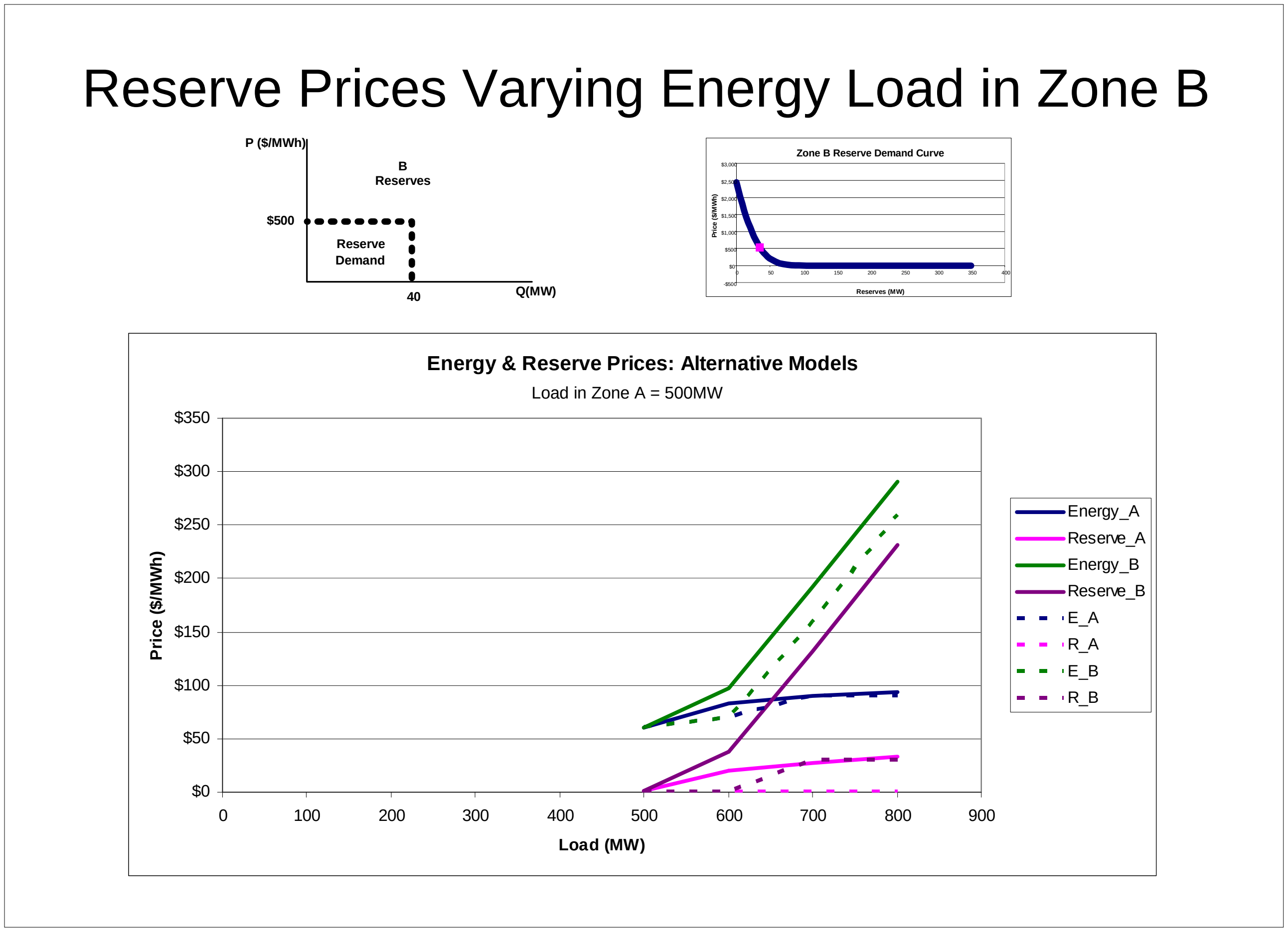
| | | | |
|---|---|---|---|
| Load A | 500 | Load B | 925 |
| | Energy | Reserves | |
| GA_1 | 581 | 19 | |
| GA_2 | 169 | 40 | |
| GB_1 | 400 | 0 | |
| GB_2 | 275 | 25 | |



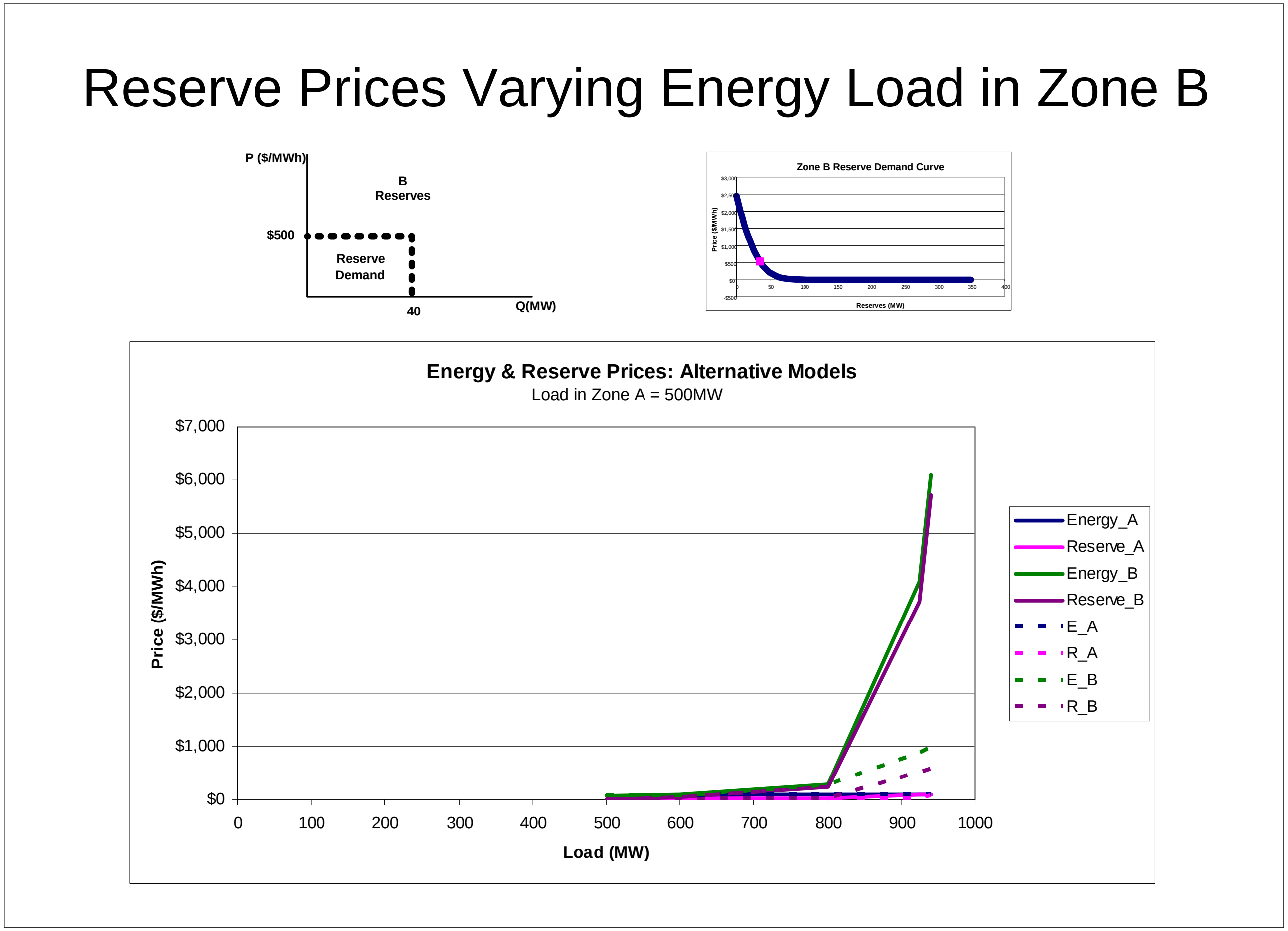



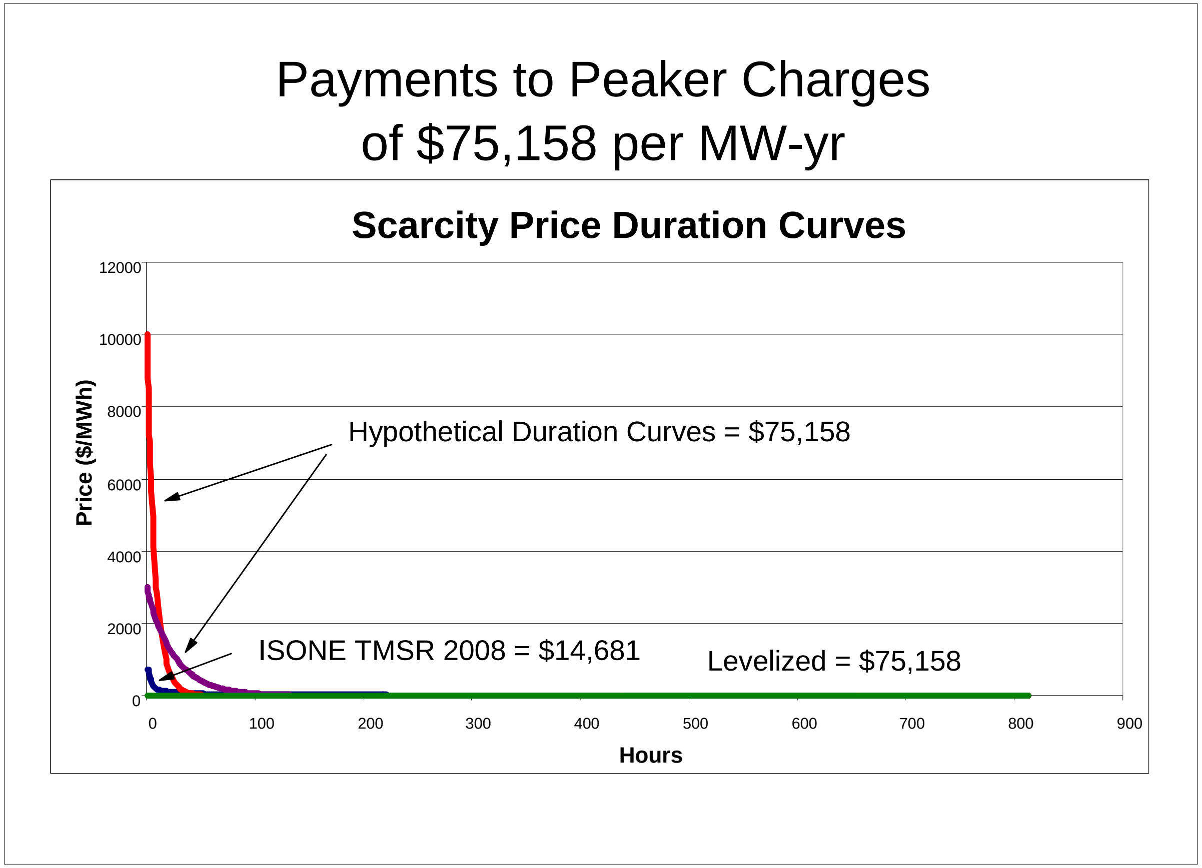
**Varying the load at Zone B illustrates the differences in energy and operating reserve locational prices for the PJM cascade assumptions and the alternative interdependent demand curves. Operating reserve prices are generally higher for the interdependent demand curves.**



**At very high loads in Zone B, the difference in scarcity prices between the alternative models is more pronounced.**



**Different scarcity pricing duration curves will determine the contribution of scarcity prices to total payments for energy and reserves. For example, consider the PJM estimate of a fixed charge for a peaker at $75,158 per MW-yr. The hypotheticals illustrate consistent alternative duration curves. These are compared with the actual 2008 price duration curve in ISONE for ten minute spinning reserves (TMSR) for location ID 7000.**



**Compared to a perfect model, there are many simplifying assumptions needed to specify and operating reserve demand curve. The sketch of the operating reserve demand curve(s) in a network could be extended.**

- **Empirical Estimation.** Use existing LOLP models or LOLP extensions with networks to estimate approximate LOLP distributions at nodes.
- **Value of Lost Load.** There are different estimates of lost load. For demand curve estimation the relevant value is the marginal of the average VOLL across the group that would first be curtailed in the event of an outage greater than the available reserves.
- **Multiple Periods.** Incorporate multiple periods of commitment and response time. Handled through the usual supply limits on ramping.
- **Operating Rules.** Incorporate up and down ramp rates, deratings, emergency procedures, etc.
- **Pricing incidence.** Charging participants for impact on operating reserve costs, with any balance included in uplift.[6]
- **Minimum Uplift Pricing.** Dispatch-based pricing that resolves inconsistencies by minimizing the total value of the price discrepancies.
- **…**

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[6] Brendan Kirby and Eric Hirst, "Allocating the Cost of Contingency Reserves," *The Electricity Journal*, December 2003, 99. 39-47.

**Supplemental material**

- **On design of operating reserve demand curve.**

**Begin with an expected value formulation of economic dispatch that might appeal in principle. Given benefit (*B*) and cost (*C*) functions, demand (*d*), generation (*g*), plant capacity (*Cap*), reserves (*r*), commitment decisions (*u*), transmission constraints (*H*), and state probabilities (*p*):**

$$\underset{y^i, d^i, g^i, r, u \in \{0,1\}}{Max} \; p_0 \left( B^0 \left( d^0 \right) - C^0 \left( g^0, r, u \right) \right) + \sum_{i=1}^{N} p_i \left( B^i \left( d^i, d^0 \right) - C^i \left( g^i, g^0, r, u \right) \right)$$

$$s.t.$$

$$y^i = d^i - g^i, \quad i = 0,1,2,\cdots,N,$$

$$\iota^t y^i = 0, \quad i = 0,1,2,\cdots,N,$$

$$H^i y^i \le b^i, \quad i = 0,1,2,\cdots,N,$$

$$g^0 + r \le u \bullet Cap^0,$$

$$g^i \le g^0 + r, \quad i = 1,2,\cdots,N,$$

$$g^i \le u \bullet Cap^i, \quad i = 0,1,2,\cdots,N.$$

Suppose there are *K* possible contingencies. The interesting cases have $K \gg 10^3$. The number of possible system states is $N = 2^K$, or more than the stars in the Milky Way. Some approximation will be in order.[7]

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[7] Shams N. Siddiqi and Martin L. Baughman, "Reliability Differentiated Pricing of Spinning Reserve," IEEE Transactions on Power Systems, Vol. 10, No. 3, August 1995, pp.1211-1218. José M. Arroyo and Francisco D. Galiana, "Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets," IEEE Transactions On Power Systems, Vol. 20, No. 2, May 2005, pp. 634-643. François Bouffard, Francisco D. Galiana, and Antonio J. Conejo, "Market-Clearing With Stochastic Security—Part I: Formulation," IEEE Transactions On Power Systems, Vol. 20, No. 4, November 2005, pp. 1818-1826; "Part II: Case Studies," pp. 1827-1835.

**Introduce random changes in load $\varepsilon^i$ and possible lost load $l^i$ in at least some conditions.**

$$\underset{y^i,g^i,l^i,r,u\in\{0,1\}}{Max}\; p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^o+\varepsilon^i-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)$$

$$\begin{aligned}
&s.t.\\
&\quad y^0=d^0-g^0,\\
&\quad y^i=d^0+\varepsilon^i-g^i-l^i,\quad i=1,2,\cdots,N,\\
&\quad \iota^t y^i=0,\quad i=0,1,2,\cdots,N,\\
&\quad H^i y^i\le b^i,\quad i=0,1,2,\cdots,N,\\
&\quad g^0+r\le u\bullet Cap^0,\\
&\quad g^i\le g^0+r,\quad i=1,2,\cdots,N,\\
&\quad g^i\le u\bullet Cap^i,\quad i=0,1,2,\cdots,N.
\end{aligned}$$

Simplify the benefit and cost functions:

$$B^i\left(d^o+\varepsilon^i-l^i,d^0\right)\approx B^0\left(d^0\right)+k_d^i-v^t l^i\quad,\qquad C^i\left(g^i,g^0,r,u\right)\approx C^0\left(g^0,r,u\right)+k_g^i\,.$$

This produces an approximate objective function:

$$p_0\left(B^0\left(d^0\right)-C^0\left(g^0,r,u\right)\right)+\sum_{i=1}^{N}p_i\left(B^i\left(d^o-l^i,d^0\right)-C^i\left(g^i,g^0,r,u\right)\right)=B^0\left(d^0\right)-C^0\left(g^0,r,u\right)+\sum_{i=1}^{N}p_i\left(k_d^i-k_g^i\right)-v^t\sum_{i=1}^{N}p_i l^i\,.$$

**The revised formulation highlights the pre-contingency objective function and the role of the value of the expected undeserved energy.**

$$\underset{y^i, g^i, l^i, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - v^t \sum_{i=1}^{N} p_i l^i$$

$$\begin{aligned}
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
&\quad \iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N, \\
&\quad H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
&\quad g^i \le u \bullet Cap^i, \quad i = 0, 1, 2, \cdots, N.
\end{aligned}$$

There are still too many system states.

**Define the optimal value of expected unserved energy (VEUE) as the result of all the possible optimal post-contingency responses given the pre-contingency commitment and scheduling decisions.**

$$VEUE\left(d^0, g^0, r, u\right) = \underset{y^i, g^i, l^i}{Min}\, v^t \sum_{i=1}^{N} p_i l^i$$

$$\begin{aligned}
&s.t. \\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
&\quad \iota^t y^i = 0, \quad i = 1, 2, \cdots, N, \\
&\quad H^i y^i \le b^i, \quad i = 1, 2, \cdots, N, \\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
&\quad g^i \le u \bullet Cap^i, \quad i = 1, 2, \cdots, N.
\end{aligned}$$

This second stage problem subsumes all the redispatch and curtailment decisions over the operating period after the commitment and scheduling decisions.

**The expected value formulation reduces to a much more manageable scale with the introduction of the implicit VEUE function.**

$$
\begin{aligned}
&\underset{y^0, d^0, g^0, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right) \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad H^0 y^0 \le b^0, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad \iota^t y^0 = 0, \\
&\quad g^0 \le u \bullet Cap^0.
\end{aligned}
$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

**The probability calculation for the constrained zone in the zonal model includes the following key element:**[8]

$$P\left(y_a + y_b \geq k_1 \middle| y_b = x_b\right) = P\left(y_a + x_b \geq k_1 \middle| y_b = x_b\right) = P\left(y_a + x_b \geq k_1\right) = 1 - F_a\left(k_1 - x_b\right)$$

$$P\left(y_a \leq x_b \middle| y_b \leq k_2\right) = F_b\left(x_b\right) / F_b\left(k_2\right)$$

$$f_{y_b | y_b \leq k_2}\left(y_b\right) = f_b\left(y_b\right) / F_b\left(k_2\right)$$

$$P\left(y_a + y_b \geq k_1 \middle| y_b \leq k_2\right) = \int_{-\infty}^{k_2} P\left(y_a + x_b \geq k_1 \middle| y_b = x_b\right) f_{y_b | y_b \leq k_2}\left(x_b\right) dx_b$$

$$= \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) / F_b\left(k_2\right) dx_b = \frac{1}{F_b\left(k_2\right)} \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$

$$P\left(y_a + y_b \geq k_1, y_b \leq k_2\right) = P\left(y_a + y_b \geq k_1 \middle| y_b \leq k_2\right) P\left(y_b \leq k_2\right)$$

$$P\left(y_a + y_b \geq k_1, y_b \leq k_2\right) = \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$

**Hence**

$$P\left(y_a + y_b \geq k_1, y_b \leq k_2\right) = \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$

$$P\left(y_0 + y_1 \geq r_0 + r_1, y_1 \leq \overline{r}_1 + r_1\right) = \int_{-\infty}^{\overline{r}_1 + r_1} \left[1 - F_0\left(r_0 + r_1 - x_1\right)\right] f_1\left(x_1\right) dx_1$$

[8] Thanks to Alberto Abadie for the probability tutorial.

**The probability calculation for the rest of system in the zonal model includes the following key element:**

$$P\left(y_a + y_b \geq k_1 \middle| y_a = x_a\right) = P\left(x_a + y_b \geq k_1 \middle| y_a = x_a\right) = P\left(x_a + y_b \geq k_1\right) = 1 - F_b\left(k_1 - x_a\right)$$

$$P\left(y_a \leq x_a \middle| y_a \geq k_2\right) = F_a\left(x_a\right) \Big/ \left[1 - F_a\left(k_2\right)\right]$$

$$f_{y_a | y_a \geq k_2}\left(y_a\right) = f_a\left(y_a\right) \Big/ \left[1 - F_a\left(k_2\right)\right]$$

$$P\left(y_a + y_b \geq k_1 \middle| y_a \geq k_2\right) = \int_{k_2}^{\infty} P\left(y_a + y_b \geq k_1 \middle| y_a = x_a\right) f_{y_a | y_a \geq k_2}\left(x_a\right) dx_a$$

$$= \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) \Big/ \left[1 - F_a\left(k_2\right)\right] dx_a = \frac{1}{1 - F_a\left(k_2\right)} \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = P\left(y_a + y_b \geq k_1 \middle| y_a \geq k_2\right) P\left(y_a \geq k_2\right)$$

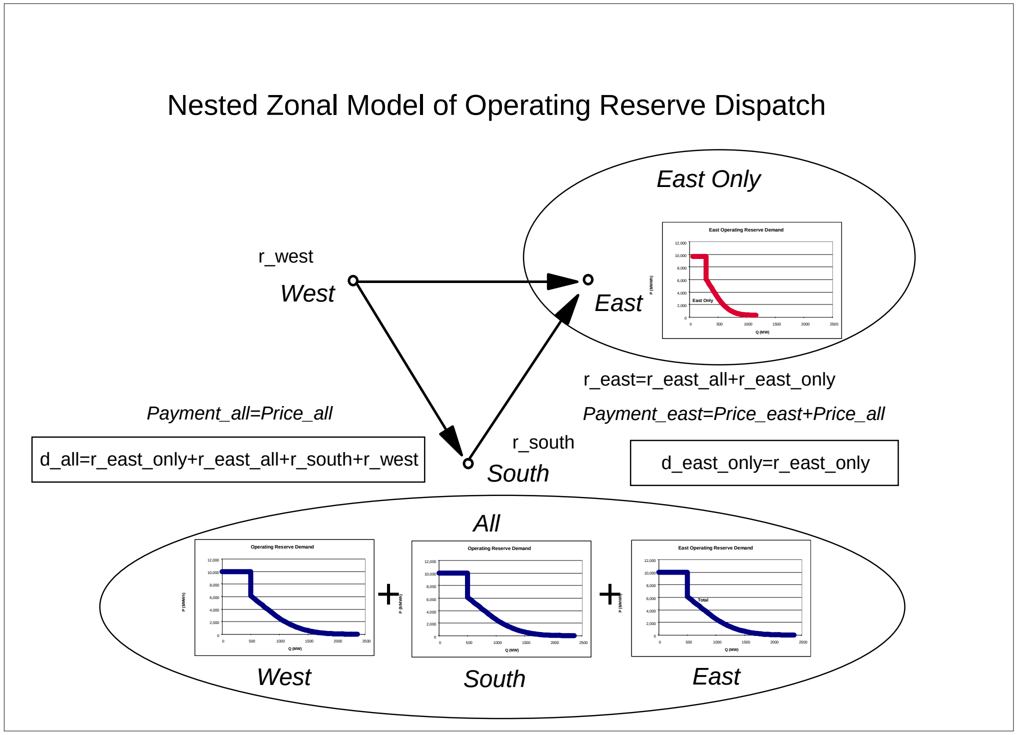
$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

**Hence.**

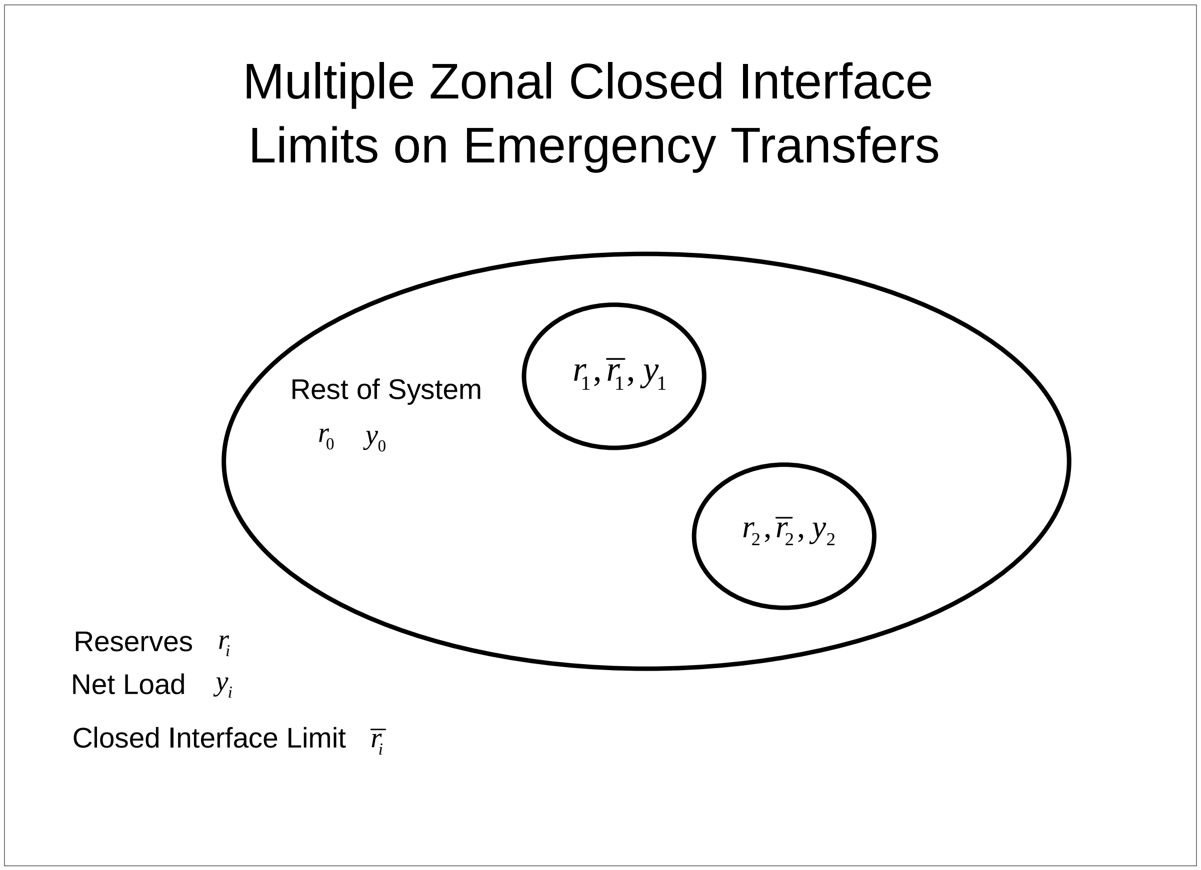
$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

$$P\left(y_0 + y_1 \geq r_0 + r_1, y_0 \geq r_0 - \overline{r}_1\right) = \int_{r_0 - \overline{r}_1}^{\infty} \left[1 - F_1\left(r_0 + r_1 - x_0\right)\right] f_0\left(x_0\right) dx_0$$

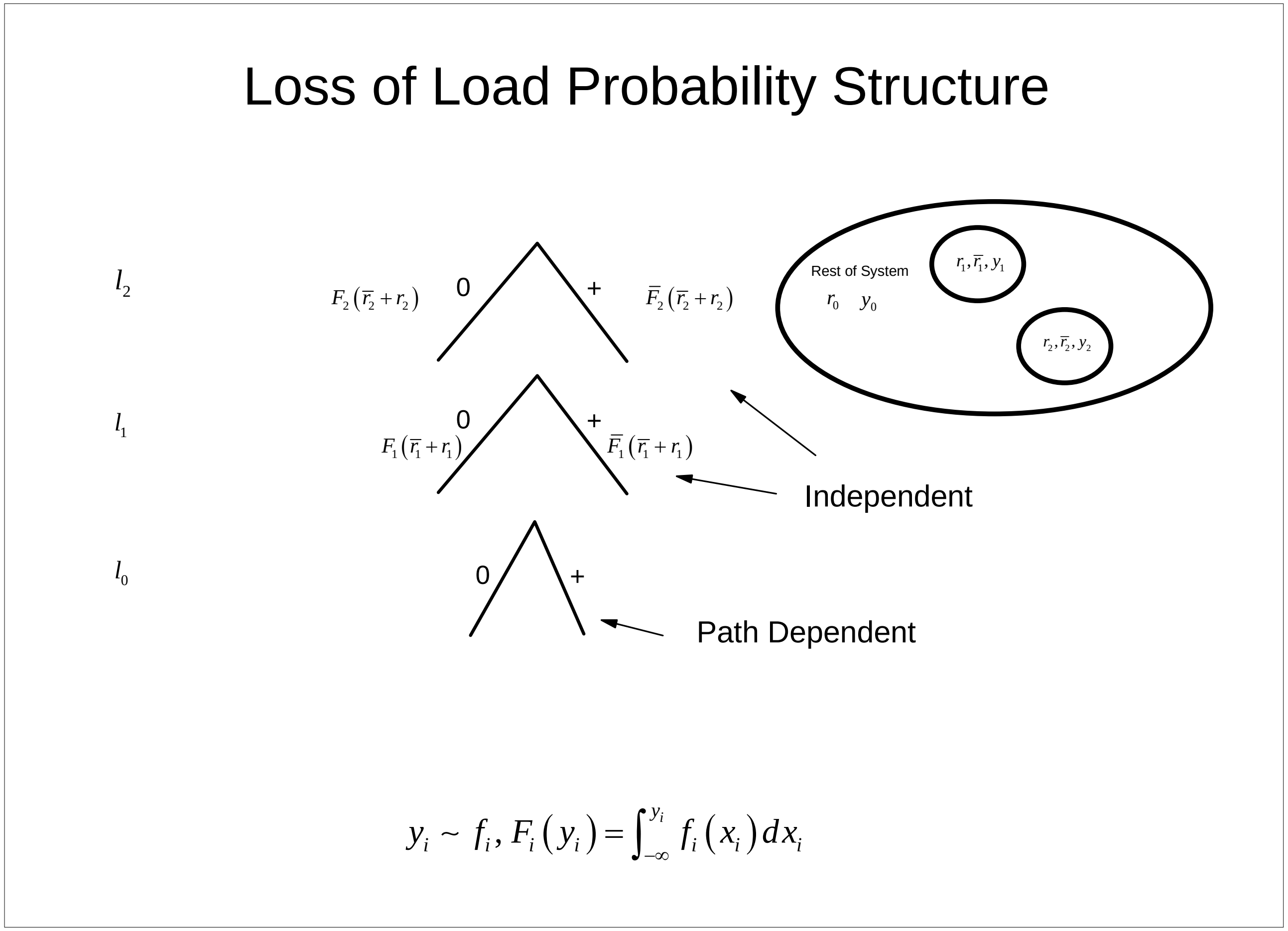
**The nested model of simultaneous dispatch of locational operating reserves and energy is used in NYISO, ISONE, and MISO. This model must derive from a different characterization of the zonal constraints. A zonal model analogous to the long-term reserve requirements approach produces interactions among regions but not in the same was as assumed in this cascade or nested formulation.**



**The case of multiple constrained zones is a natural extension of the case for a single constrained zone.**



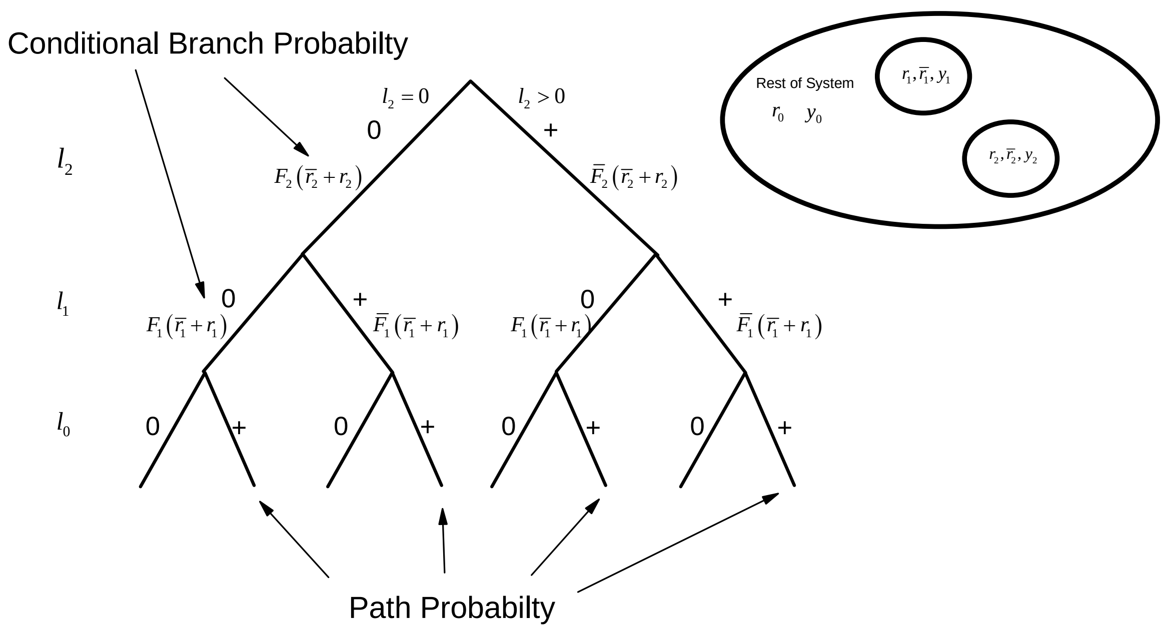
**The probability of losses depends on the path of binding interface constraints.**



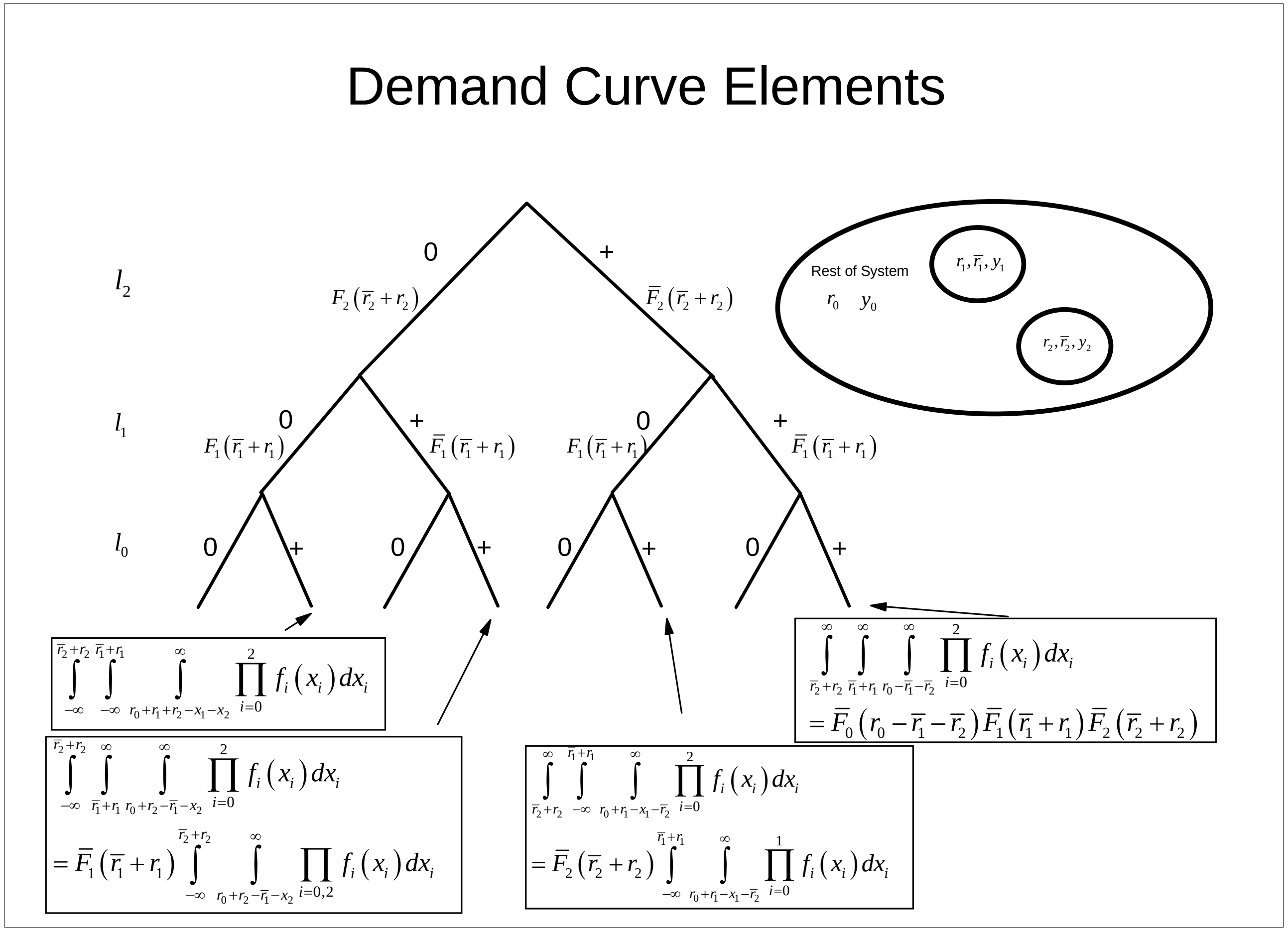
**The probability tree captures the dependencies of loss of load.**

## Loss of Load Probabilities

$$ZVEUE\left(r_0,\bar{r}_1,r_1,\bar{r}_2,r_2\right)=E_y\left[\underset{l_i\geq 0}{Min}\ \left\{v_0l_0+v_1l_1+v_2l_2 \middle| y_0+y_1+y_2-l_0-l_1-l_2\leq r_0+r_1+r_2,\ y_1-l_1\leq \bar{r}_1+r_1,\ y_2-l_2\leq \bar{r}_2+r_2\right\}\right]$$



**The loss of load probability structure defines the demand curve elements.**



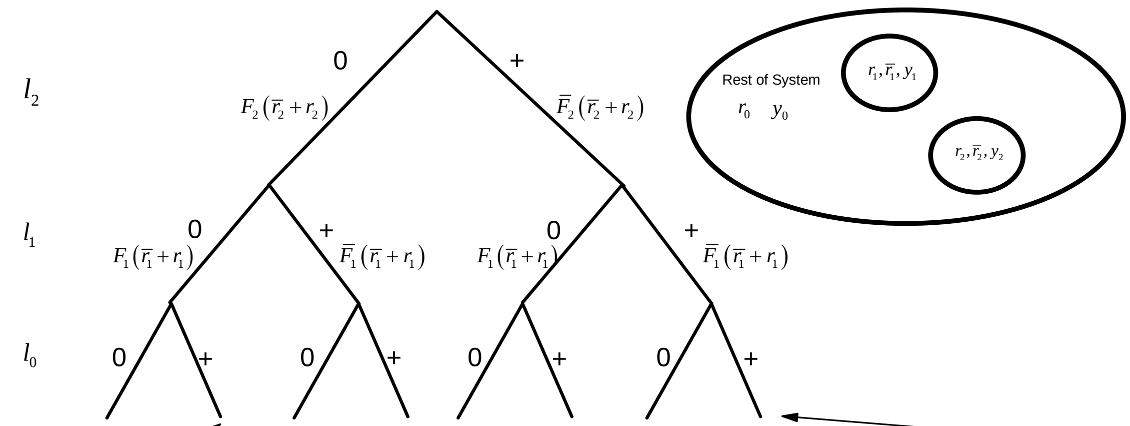
**The tree structure identifies the loss probability dependencies and the paths where incremental capacity affects the losses.**

- **Outages and Demand Changes.** The zonal convolutions of capacity outages and demand changes determine the (assumed independent) elementary zonal probability distributions of changes in net load.
- **Tree Structure.** The dependencies for losses and binding interface constraints defined by the probability tree structure determine the path probabilities for loss of load in each location as a function of the underlying independent elementary distributions.
- **Demand Curve.** The demand curve is determined by the value of lost load in each zone and the dependencies in the tree structure determining when reserves or interface capacity would be substitutable for losses.
    - **Value of Loss Load.** Assume embedded zones have higher incremental values of lost load.
    - **Substitution of Capacity.** Identify substitution possibilities on alternative paths for zonal losses and binding constraints. For example:
        - **Zonal Losses.** Apply only when interface constraint is binding.
        - **Reserve Substitution.** Higher level reserves substitute for lower level losses only when interface constraint is not binding.
        - **Interface Capacity.** Increased interface capacity for binding interface substitutes lower level losses for higher level losses.

**The loss outcomes determine demand for rest of system operating reserve.**

## Demand Curve Elements: Rest of System

$$p_{r_0} = v_o \left[ \int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \overline{F}_1(\overline{r}_1+r_1) \int_{-\infty}^{\overline{r}_2+r_2} \int_{r_0+r_2-\overline{r}_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i + \overline{F}_2(\overline{r}_2+r_2) \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i + \overline{F}_0(r_0-\overline{r}_1-\overline{r}_2)\overline{F}_1(\overline{r}_1+r_1)\overline{F}_2(\overline{r}_2+r_2) \right]$$



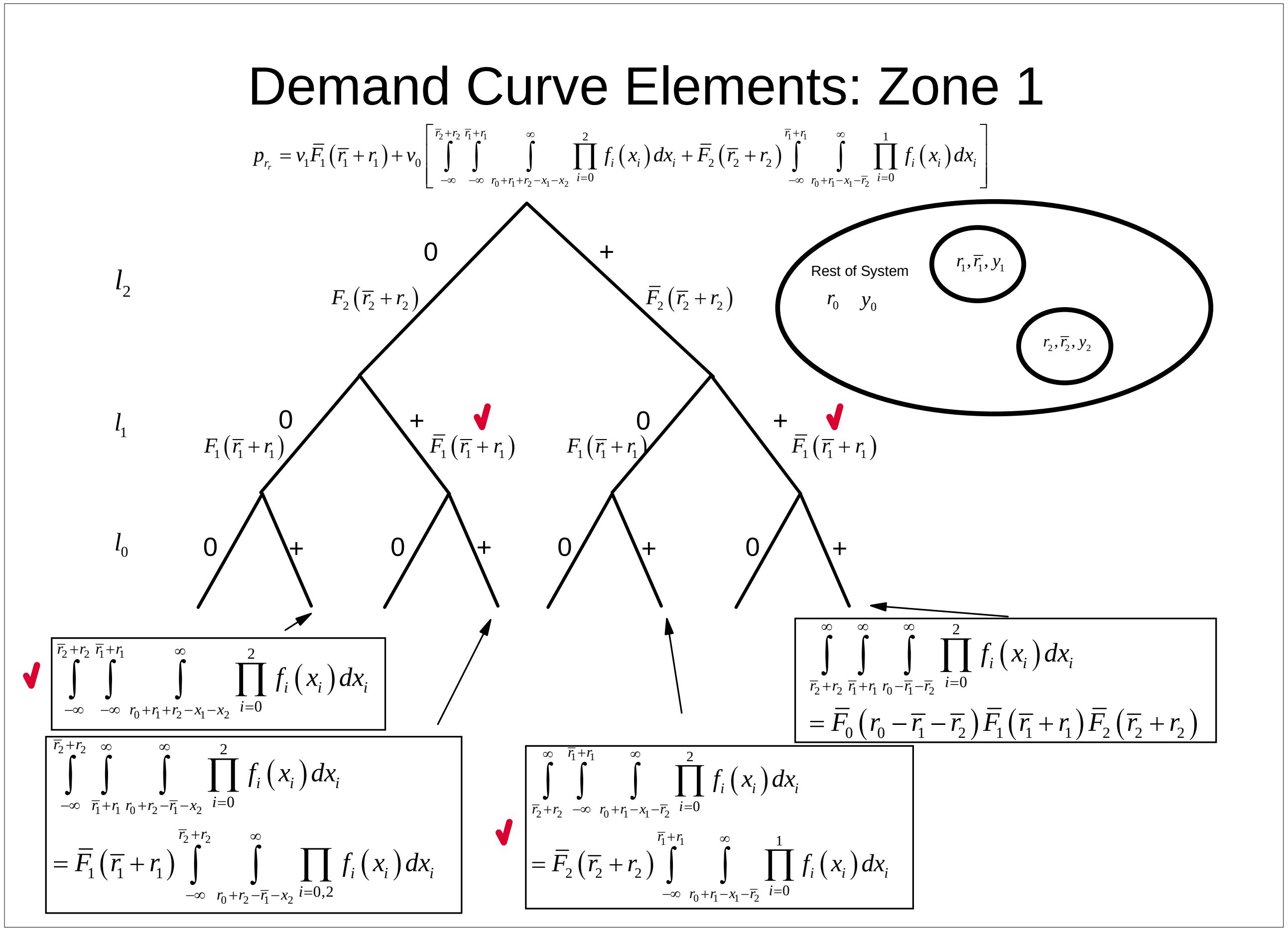

$$\int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i$$

$$\int_{\overline{r}_2+r_2}^{\infty} \int_{\overline{r}_1+r_1}^{\infty} \int_{r_0-\overline{r}_1-\overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_0(r_0-\overline{r}_1-\overline{r}_2)\overline{F}_1(\overline{r}_1+r_1)\overline{F}_2(\overline{r}_2+r_2)$$

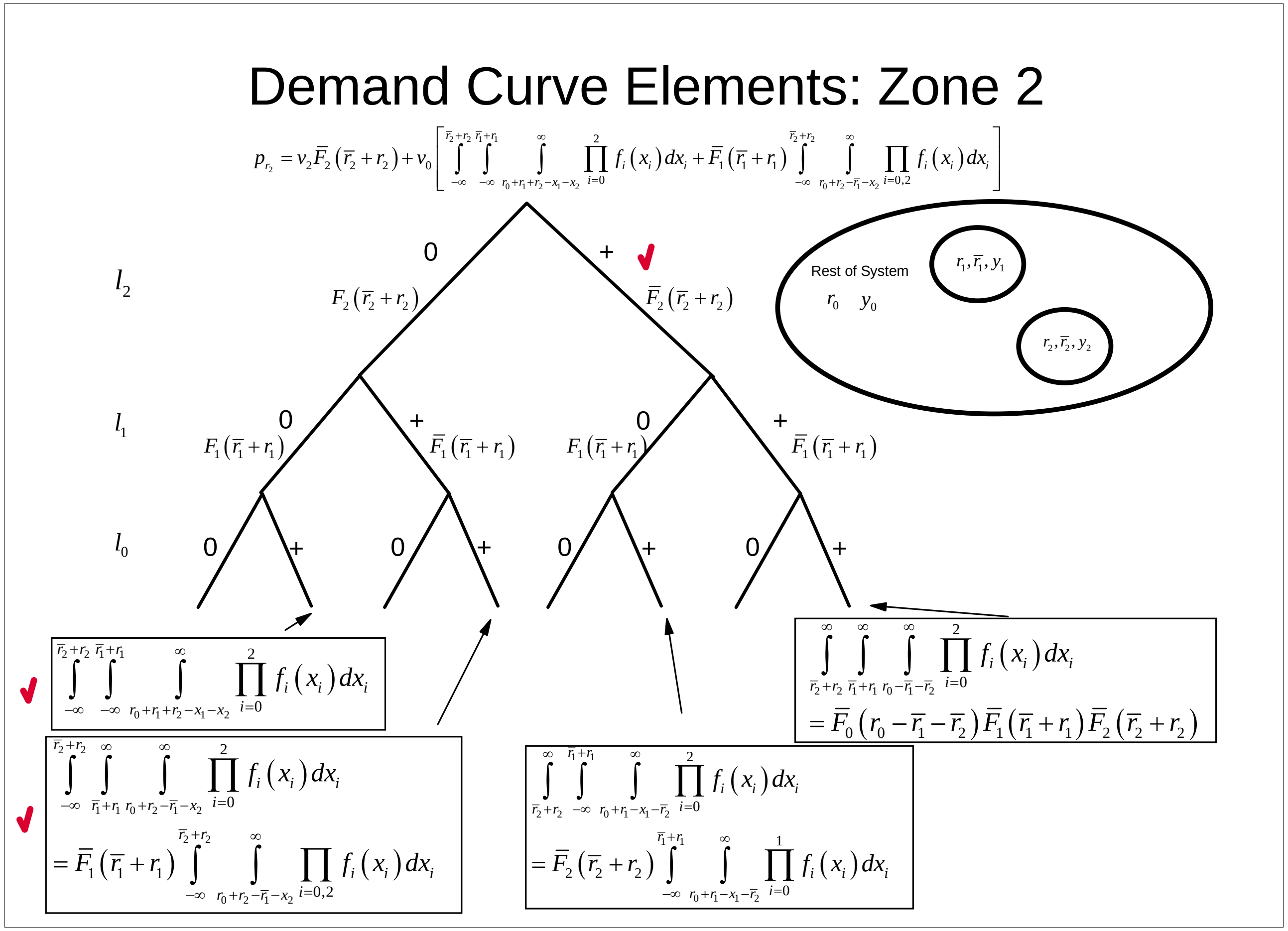
$$\int_{-\infty}^{\overline{r}_2+r_2} \int_{\overline{r}_1+r_1}^{\infty} \int_{r_0+r_2-\overline{r}_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_1(\overline{r}_1+r_1) \int_{-\infty}^{\overline{r}_2+r_2} \int_{r_0+r_2-\overline{r}_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i$$

$$\int_{\overline{r}_2+r_2}^{\infty} \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_2(\overline{r}_2+r_2) \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1-x_1-\overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i$$

**The loss outcomes and dependencies determine the demand for zone 1 operating reserves.**



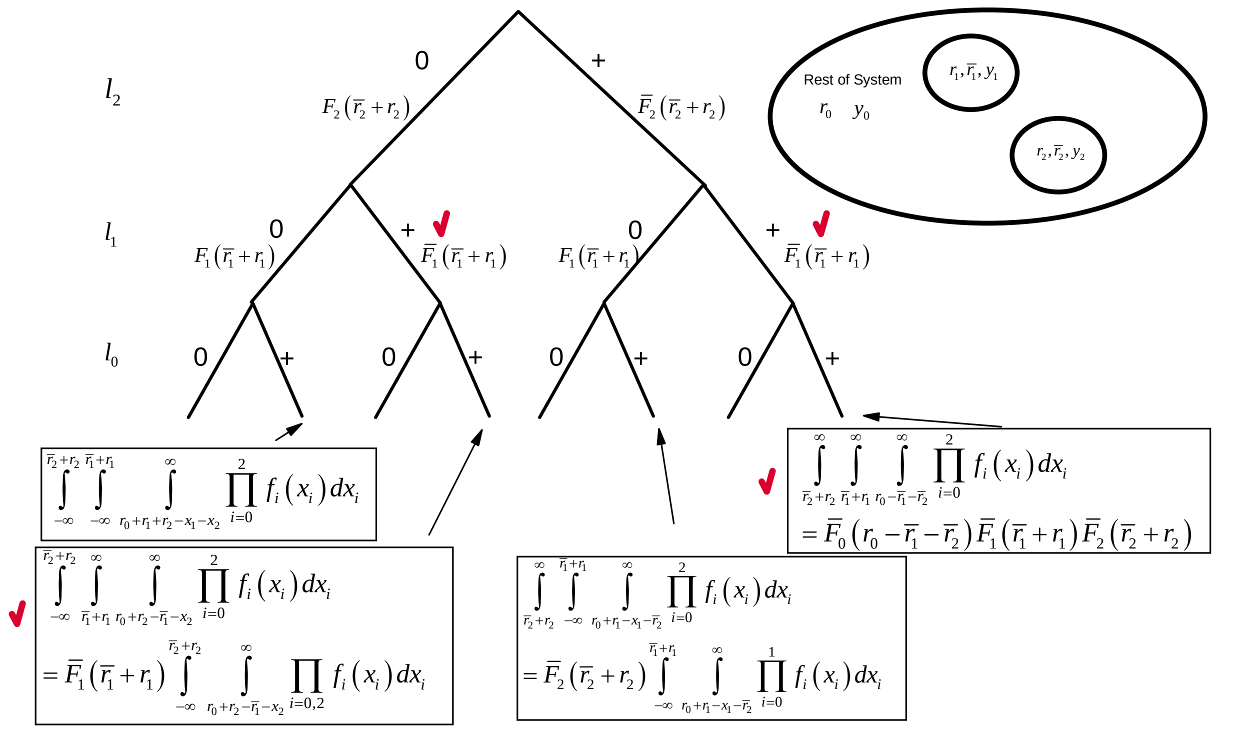
**The loss outcomes and dependencies determine the demand for zone 2 operating reserves.**



**The loss outcomes and dependencies determine the demand for zone 1 interface capacity.**

# Demand Curve Elements: Interface 1

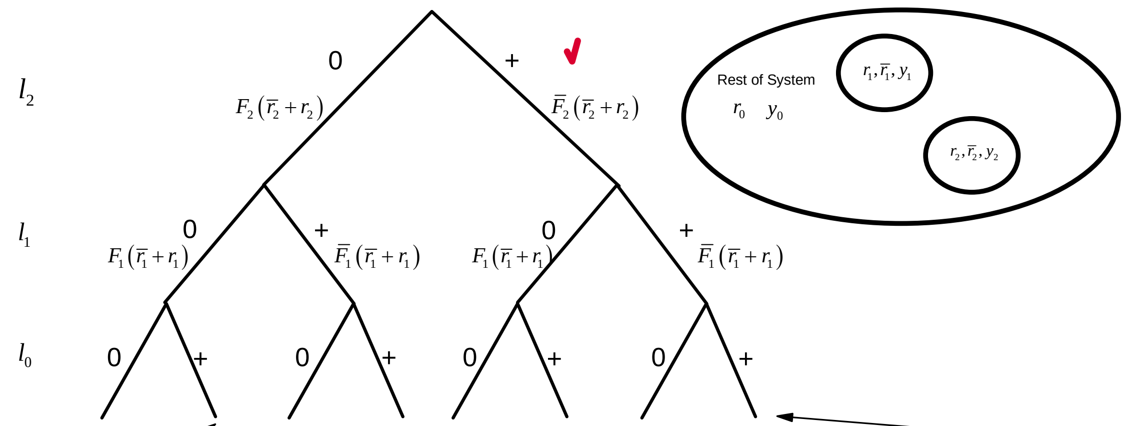
$$p_{\bar r_1} = v_1 \bar F_1(\bar r_1 + r_1) - v_0 \left[ \bar F_1(\bar r_1+r_1) \int_{-\infty}^{\bar r_2+r_2} \int_{r_0+r_2-\bar r_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1-\bar r_2)\,\bar F_1(\bar r_1+r_1)\,\bar F_2(\bar r_2+r_2)\right]$$



**The loss outcomes and dependencies determine the demand for zone 2 interface capacity.**

# Demand Curve Elements: Interface 2

$$p_{\overline{r}_2} = v_2 \overline{F}_2\left(\overline{r}_2 + r_2\right) - v_0 \left[ \overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i + \overline{F}_0\left(r_0 - \overline{r}_1 - \overline{r}_2\right) \overline{F}_1\left(\overline{r}_1 + r_1\right) \overline{F}_2\left(\overline{r}_2 + r_2\right) \right]$$



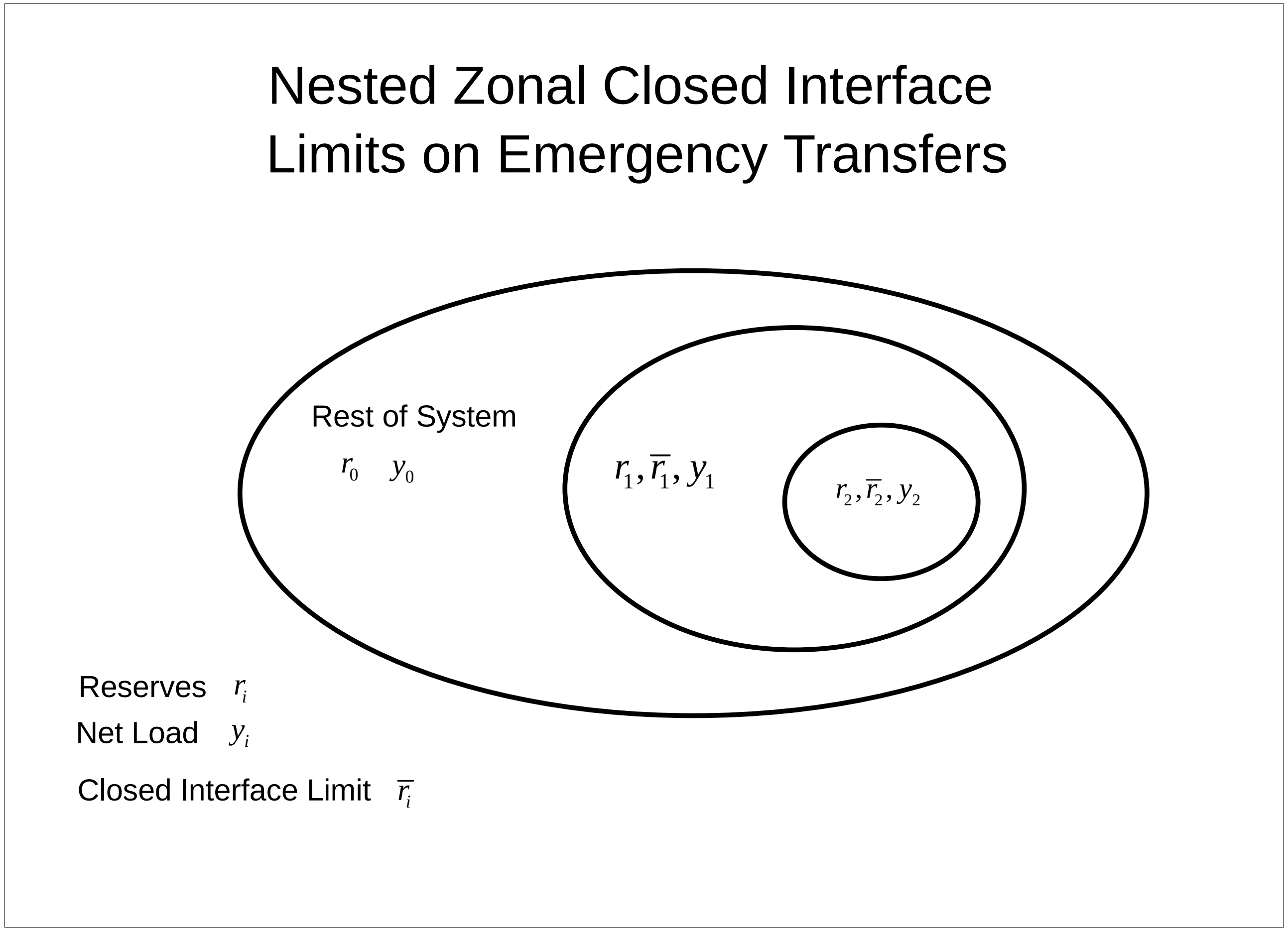

$$\int_{-\infty}^{\overline{r}_2 + r_2} \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 + r_2 - x_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$

$$\int_{-\infty}^{\overline{r}_2 + r_2} \int_{\overline{r}_1 + r_1}^{\infty} \int_{r_0 + r_2 - \overline{r}_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_1\left(\overline{r}_1 + r_1\right) \int_{-\infty}^{\overline{r}_2 + r_2} \int_{r_0 + r_2 - \overline{r}_1 - x_2}^{\infty} \prod_{i=0,2} f_i\left(x_i\right) dx_i$$

$$\int_{\overline{r}_2 + r_2}^{\infty} \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_2\left(\overline{r}_2 + r_2\right) \int_{-\infty}^{\overline{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i$$

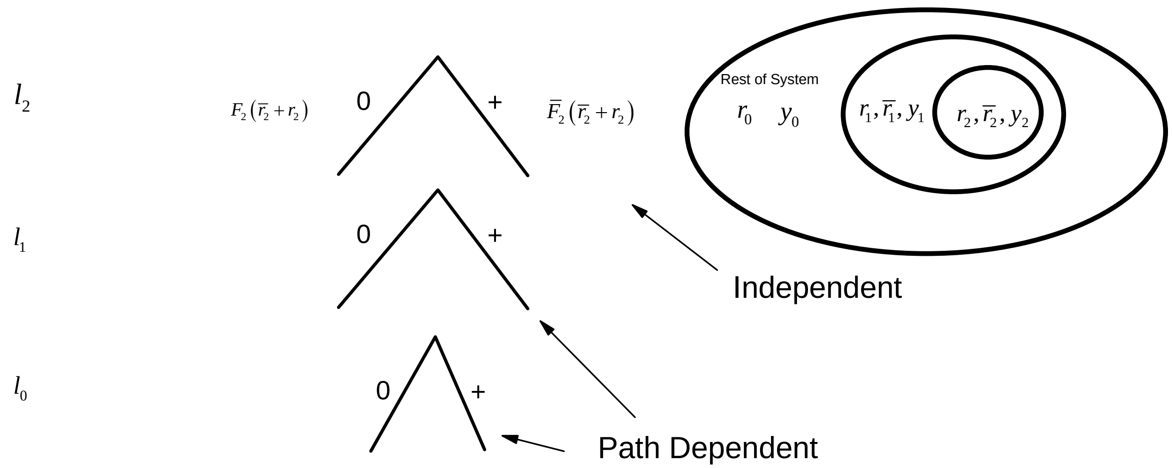
$$\int_{\overline{r}_2 + r_2}^{\infty} \int_{\overline{r}_1 + r_1}^{\infty} \int_{r_0 - \overline{r}_1 - \overline{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \overline{F}_0\left(r_0 - \overline{r}_1 - \overline{r}_2\right) \overline{F}_1\left(\overline{r}_1 + r_1\right) \overline{F}_2\left(\overline{r}_2 + r_2\right)$$

**Nested constrained zones define an alternative extension of the case for a single constrained zone.**



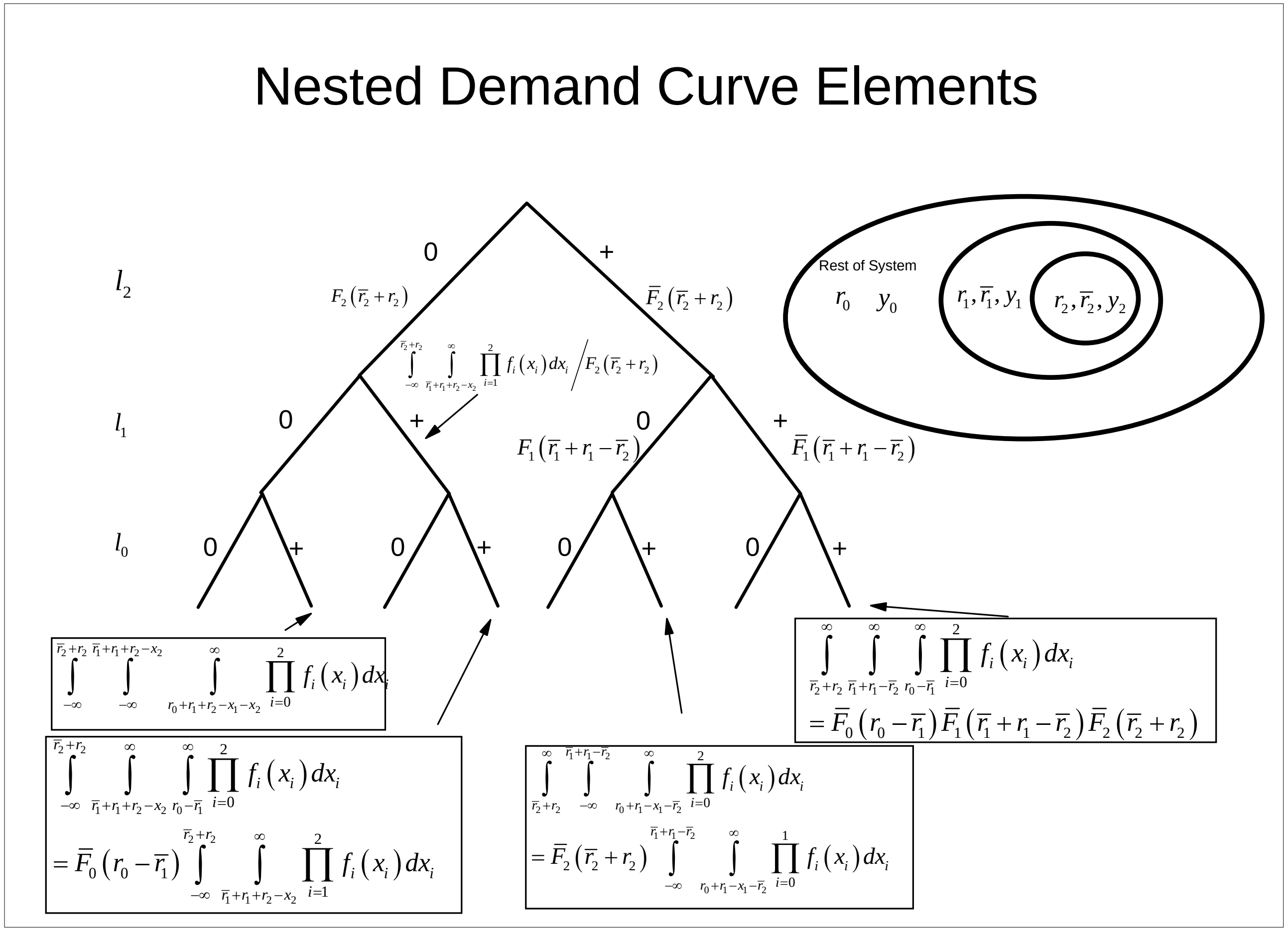
**The probability tree for the nested zones captures the dependencies of loss of load.**

## Nested Loss of Load Probability Structure




$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

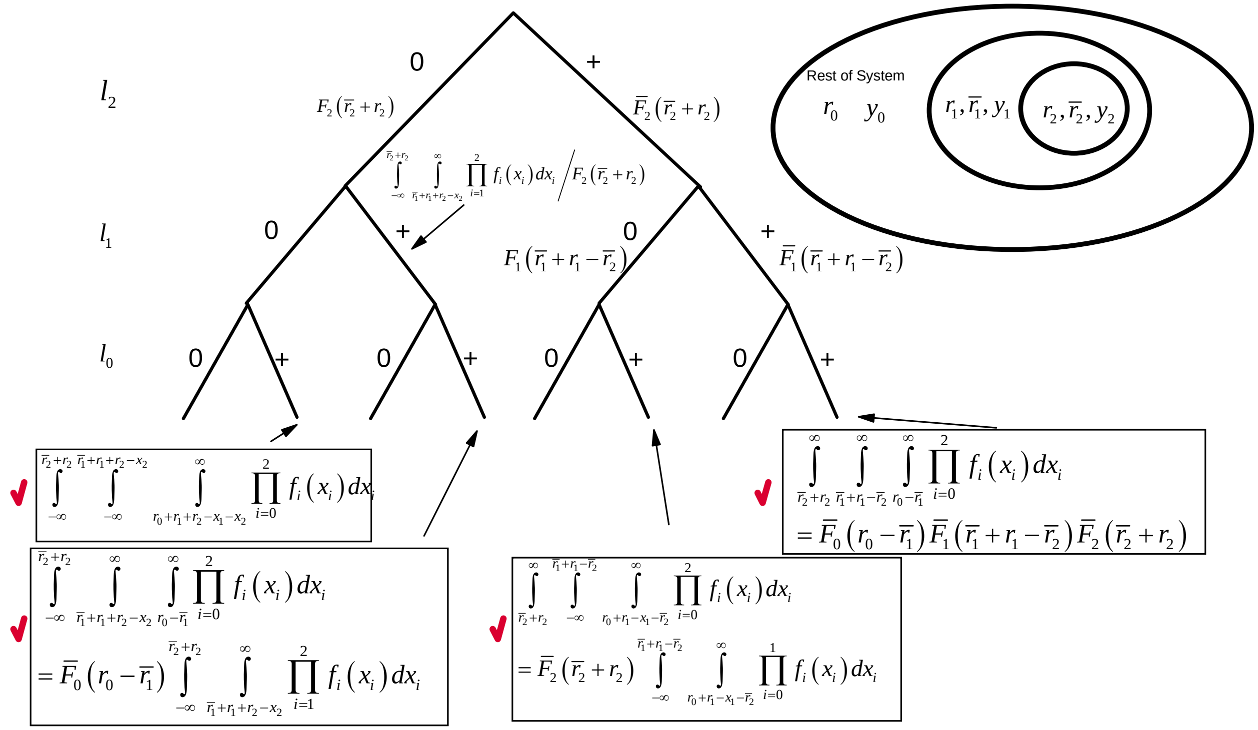
**The nested loss of load probability structure defines the demand curve elements.**



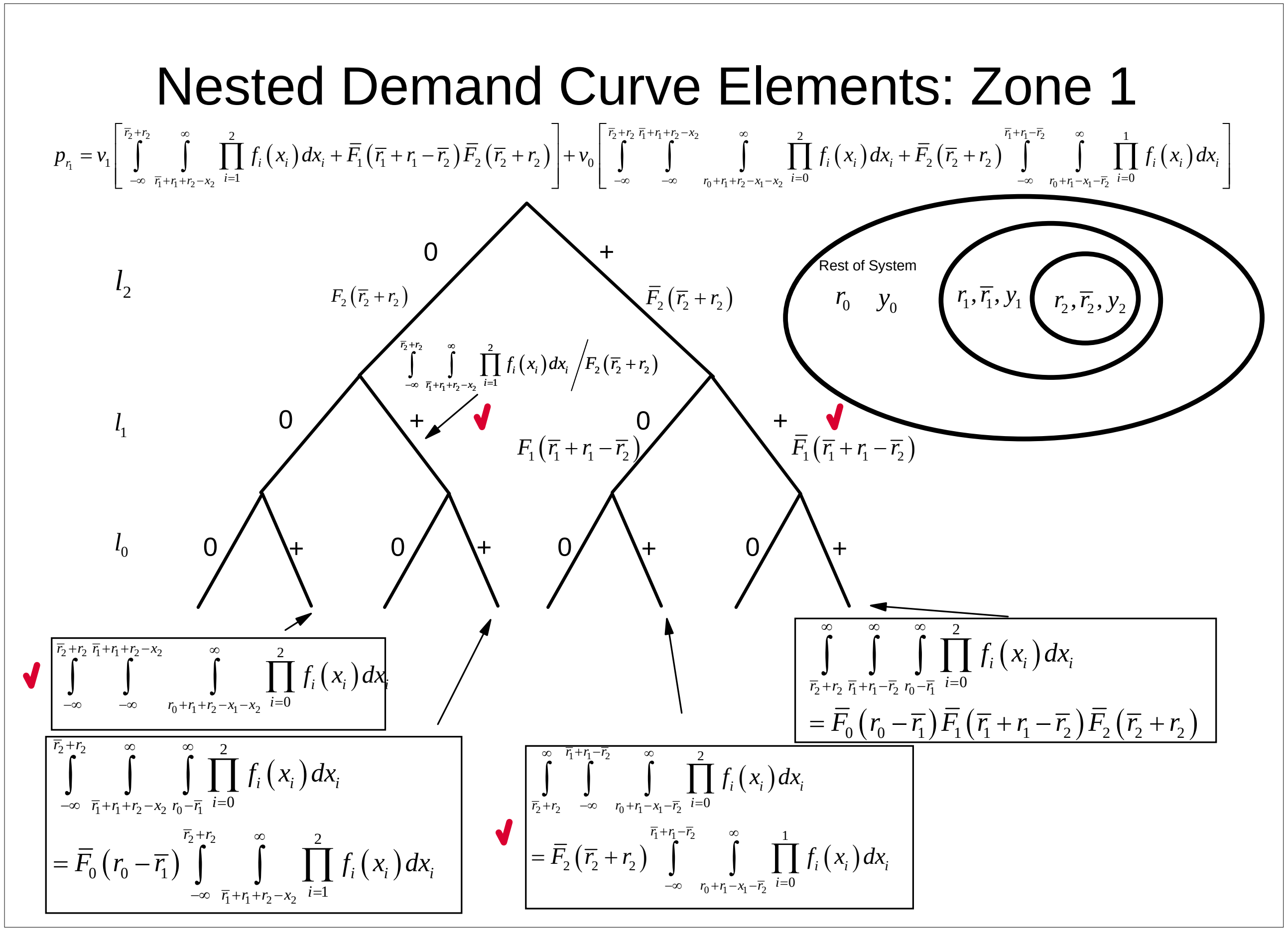
**The nested loss outcomes and dependencies determine the demand for rest of system operating reserves.**

## Nested Demand Curve Elements: Rest of System

$$p_{r_0} = v_0 \left[ \int_{-\infty}^{\bar{r}_2+r_2} \int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar{F}_0(r_0-\bar{r}_1) \int_{-\infty}^{\bar{r}_2+r_2} \int_{\bar{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar{F}_2(\bar{r}_2+r_2) \int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2} \int_{r_0+r_1-x_1-\bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i + \bar{F}_0(r_0-\bar{r}_1)\bar{F}_1(\bar{r}_1+r_1-\bar{r}_2)\bar{F}_2(\bar{r}_2+r_2) \right]$$



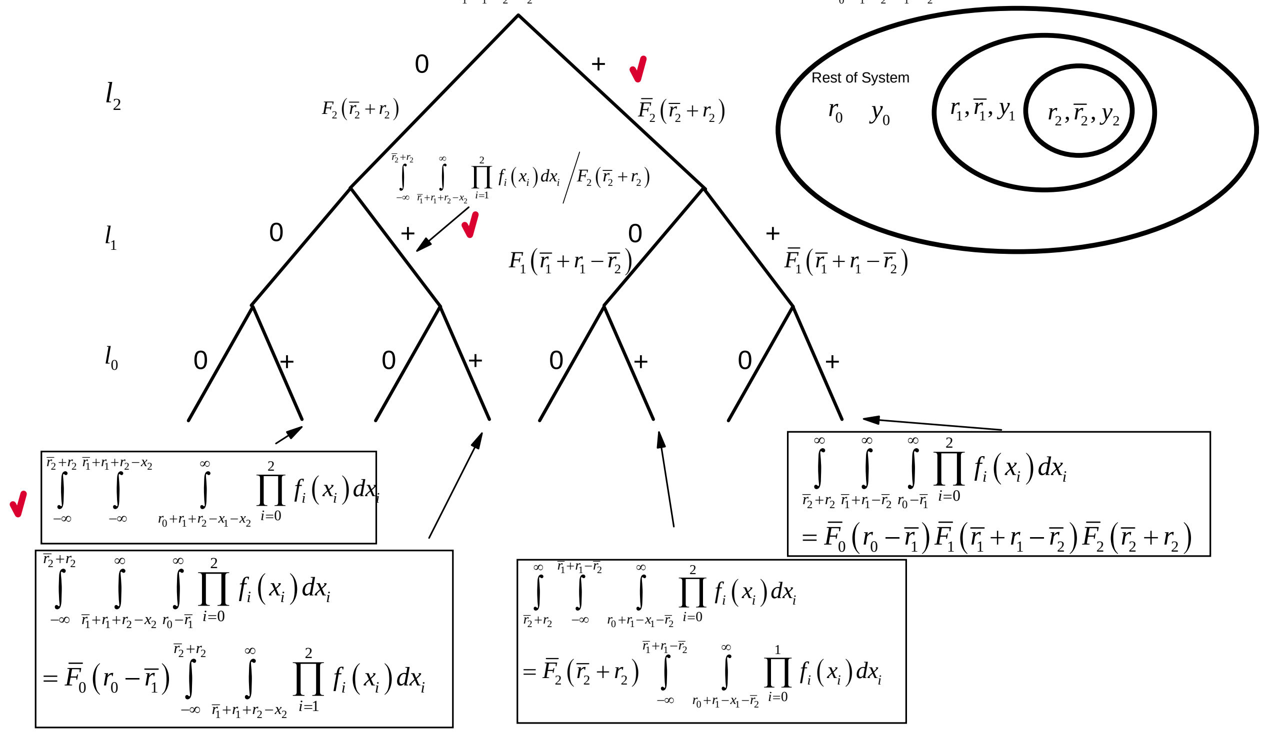
**The nested loss outcomes and dependencies determine the demand for zone 1 operating reserves.**



**The nested loss outcomes and dependencies determine the demand for zone 2 operating reserves.**

## Nested Demand Curve Elements: Zone 2

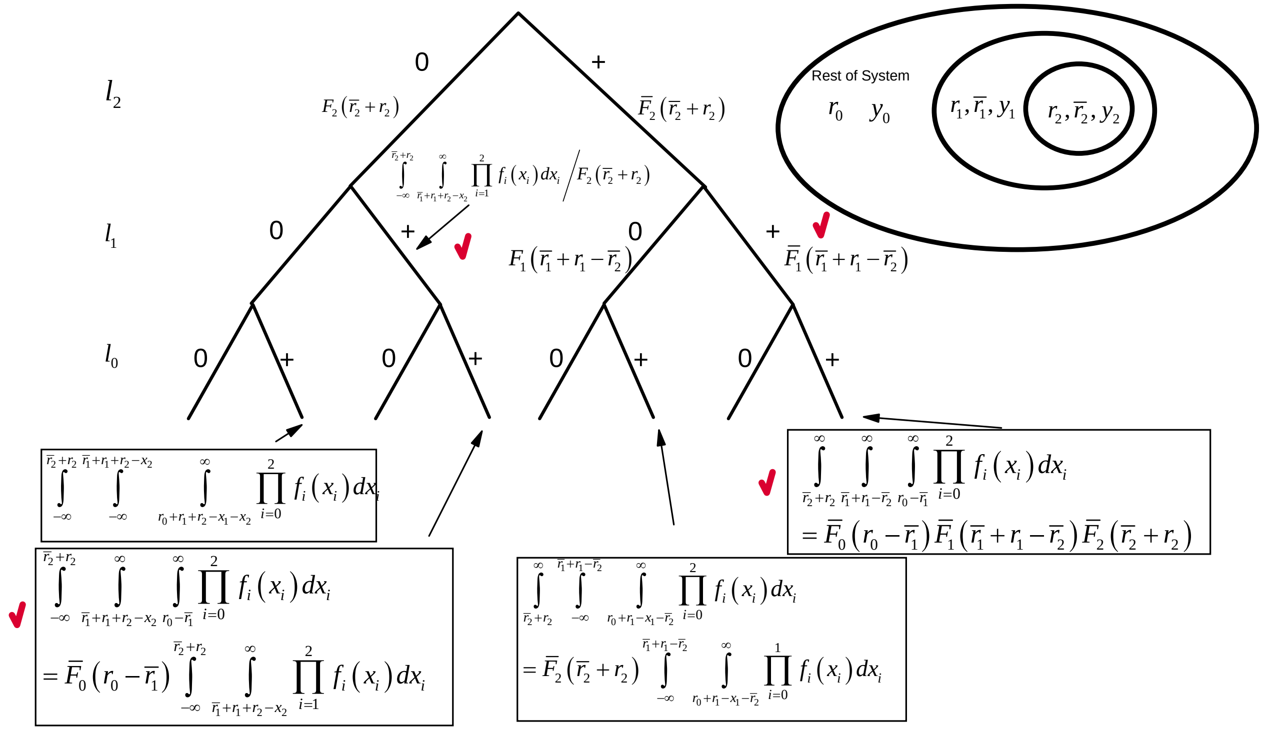
$$p_{r_2} = v_2\bar{F}_2(\bar{r}_2 + r_2) + v_1 \int_{-\infty}^{\bar{r}_2+r_2}\int_{\bar{r}_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + v_0 \int_{-\infty}^{\bar{r}_2+r_2}\int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i$$



**The nested loss outcomes and dependencies determine the demand for interface 1 capacity.**

## Nested Demand Curve Elements: Interface 1

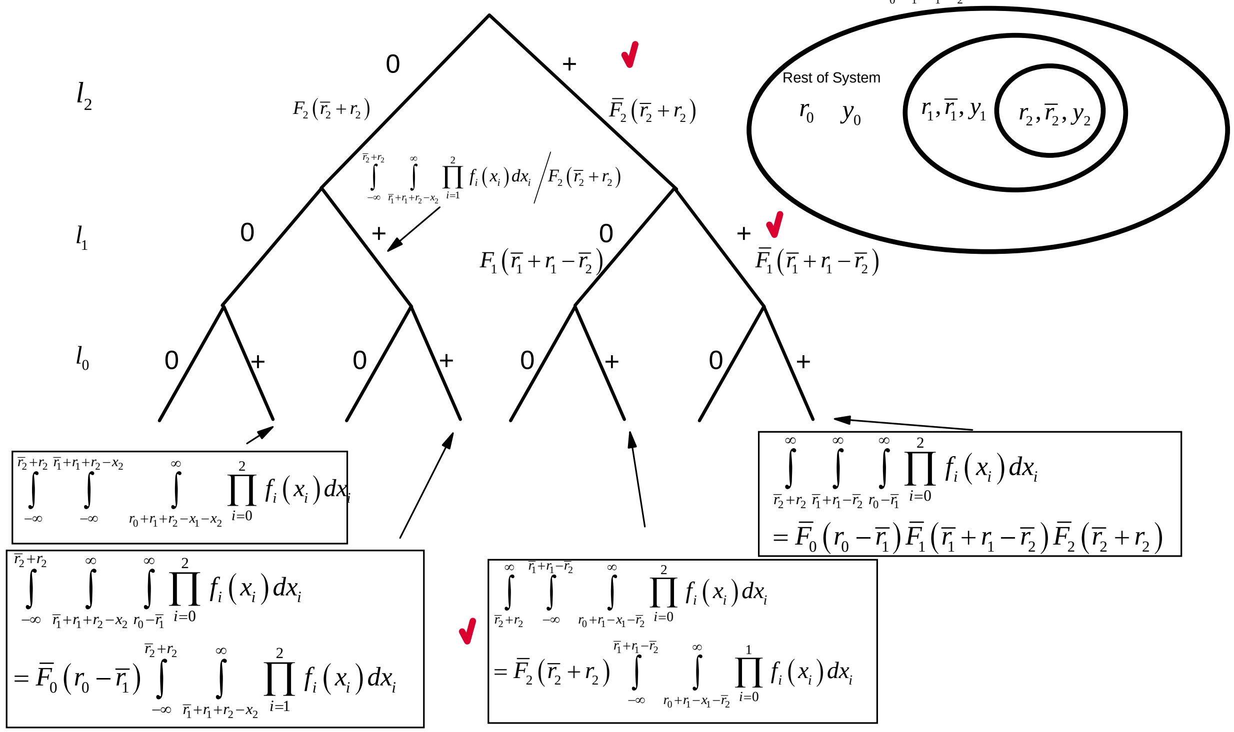
$$p_{\bar r_1} = v_1\left[\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right] - v_0\left[\bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right]$$



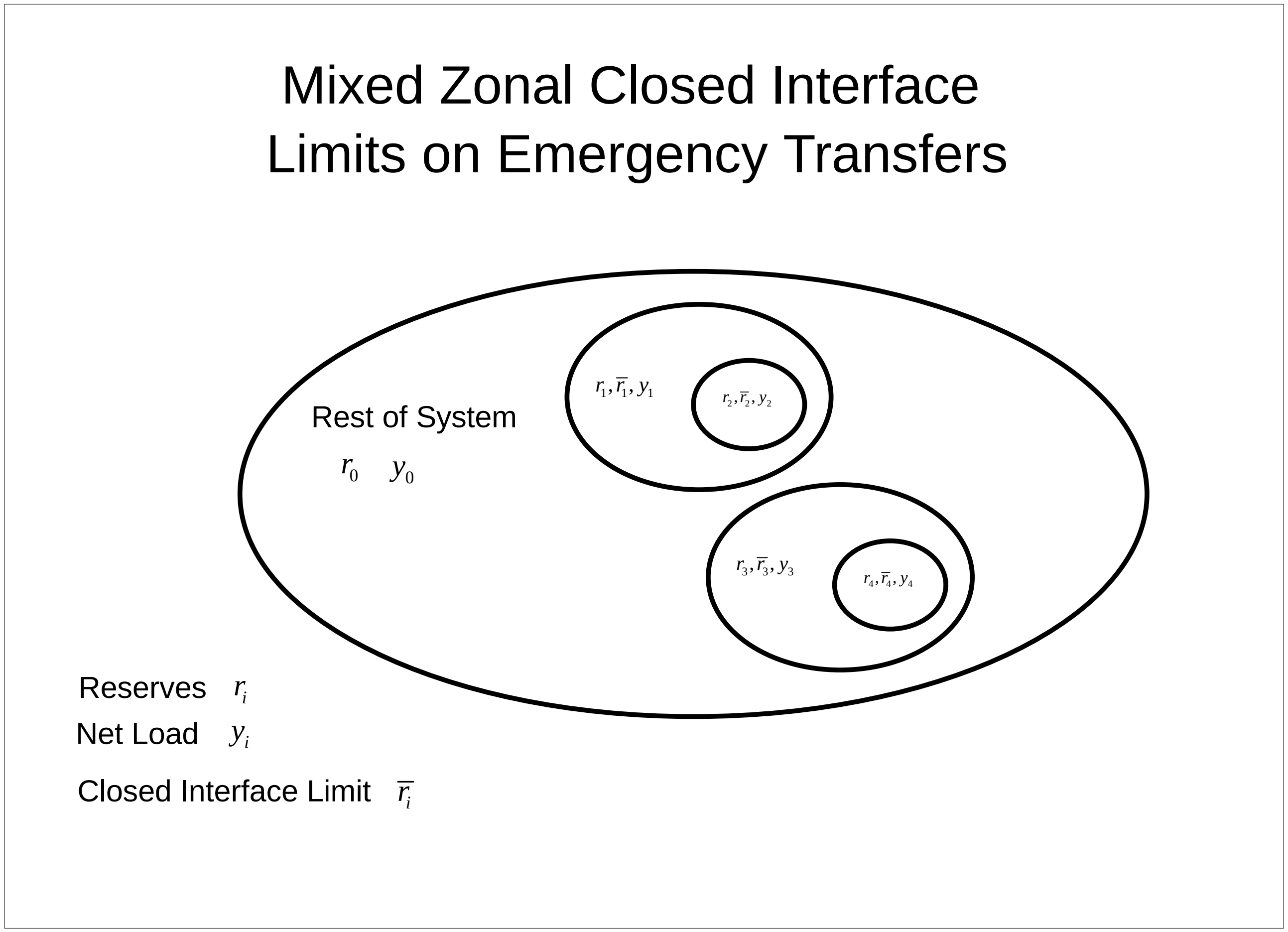
**The nested loss outcomes and dependencies determine the demand for interface 2 capacity.**

## Nested Demand Curve Elements: Interface 2

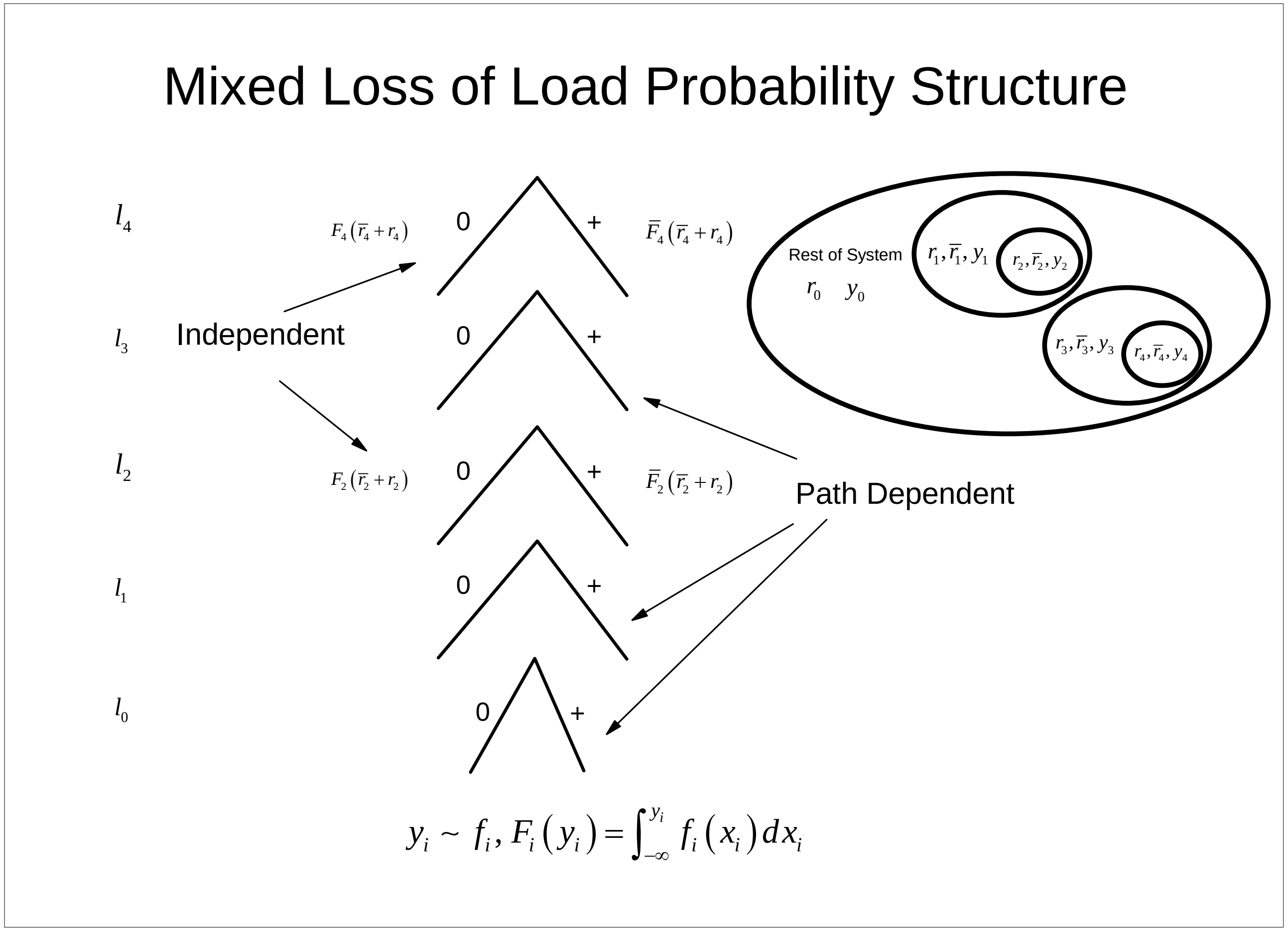
$$p_{\bar{r}_2} = v_2\bar{F}_2(\bar{r}_2 + r_2) - v_1\bar{F}_1(\bar{r}_1 + r_1 - \bar{r}_2)\bar{F}_2(\bar{r}_2 + r_2) - v_0\bar{F}_2(\bar{r}_2 + r_2)\int_{-\infty}^{\bar{r}_1 + r_1 - \bar{r}_2}\int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$



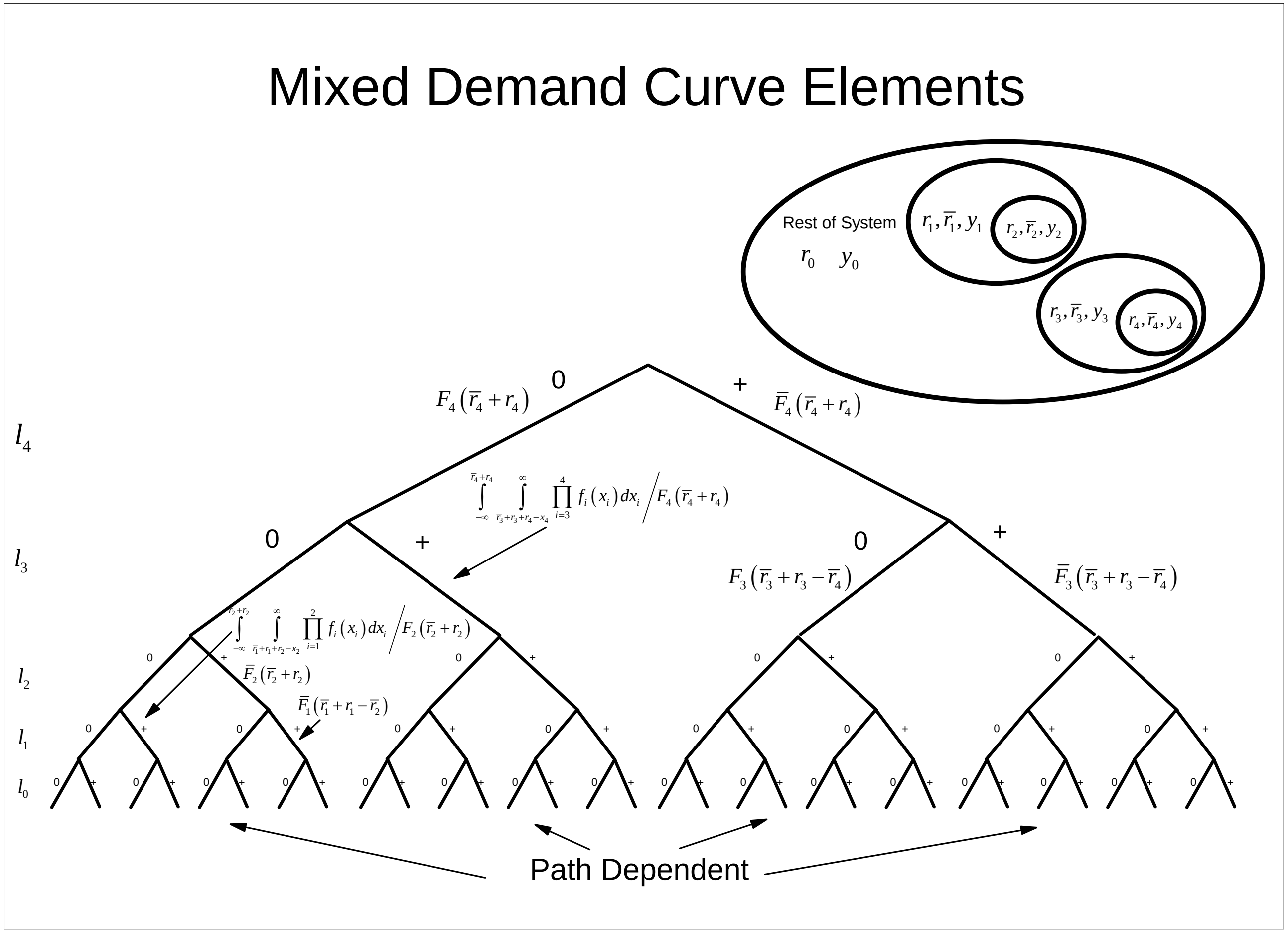
**Mixed constrained zones define a more general extension of a constrained zonal structure.**



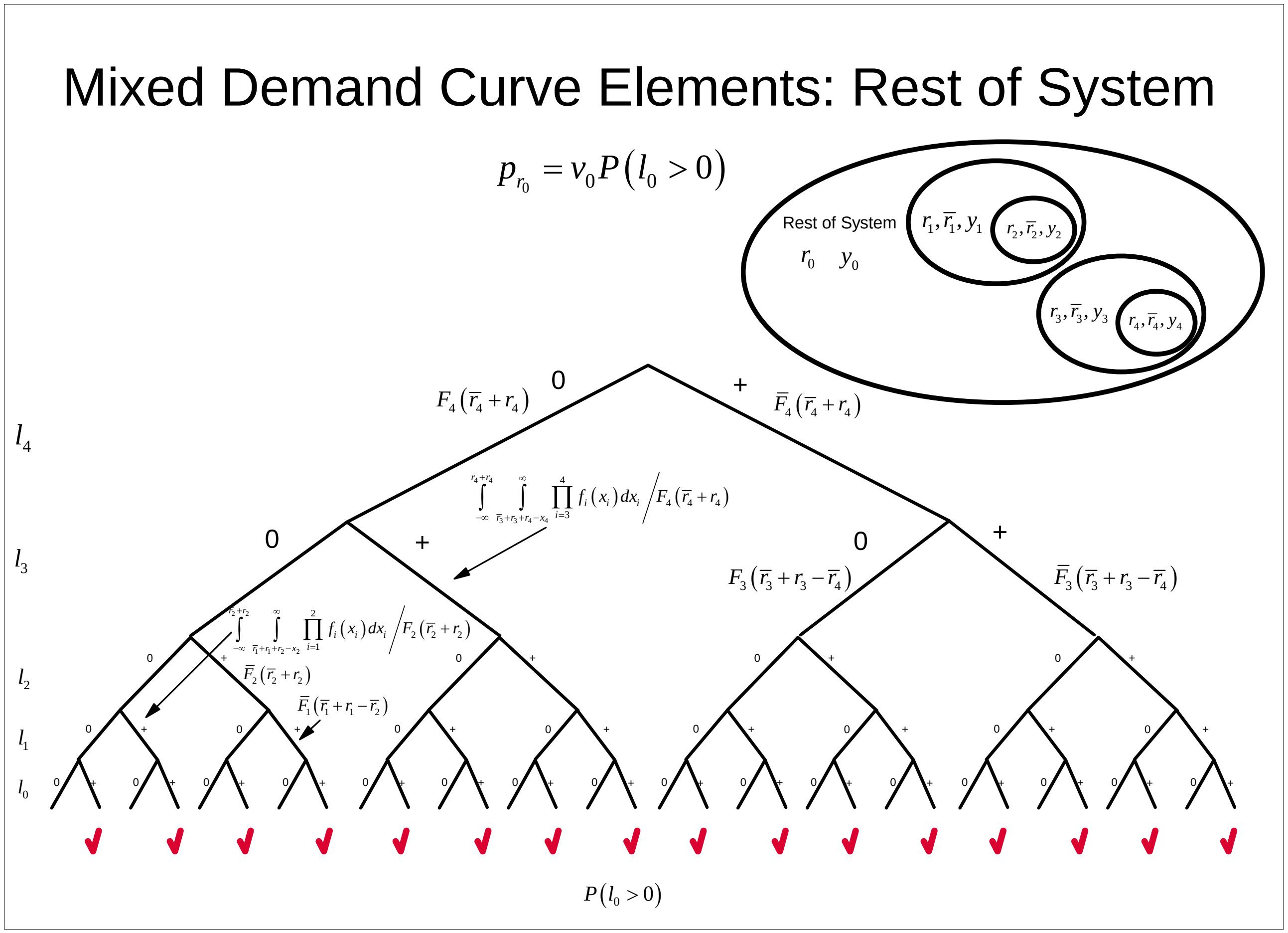
**The mixed probability tree for the nested zones captures the dependencies of loss of load.**



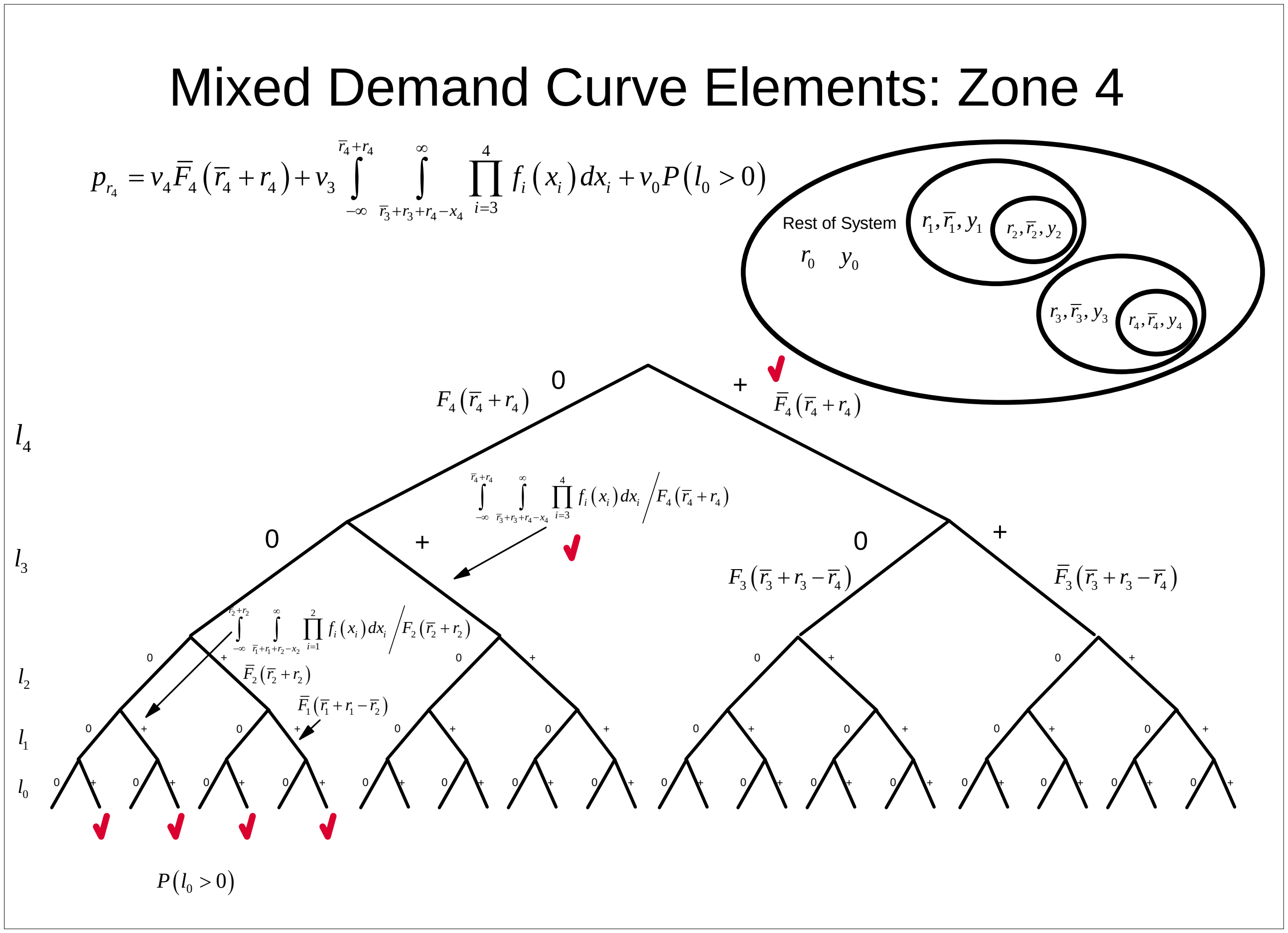
**The mixed loss of load probability structure defines the demand curve elements.**



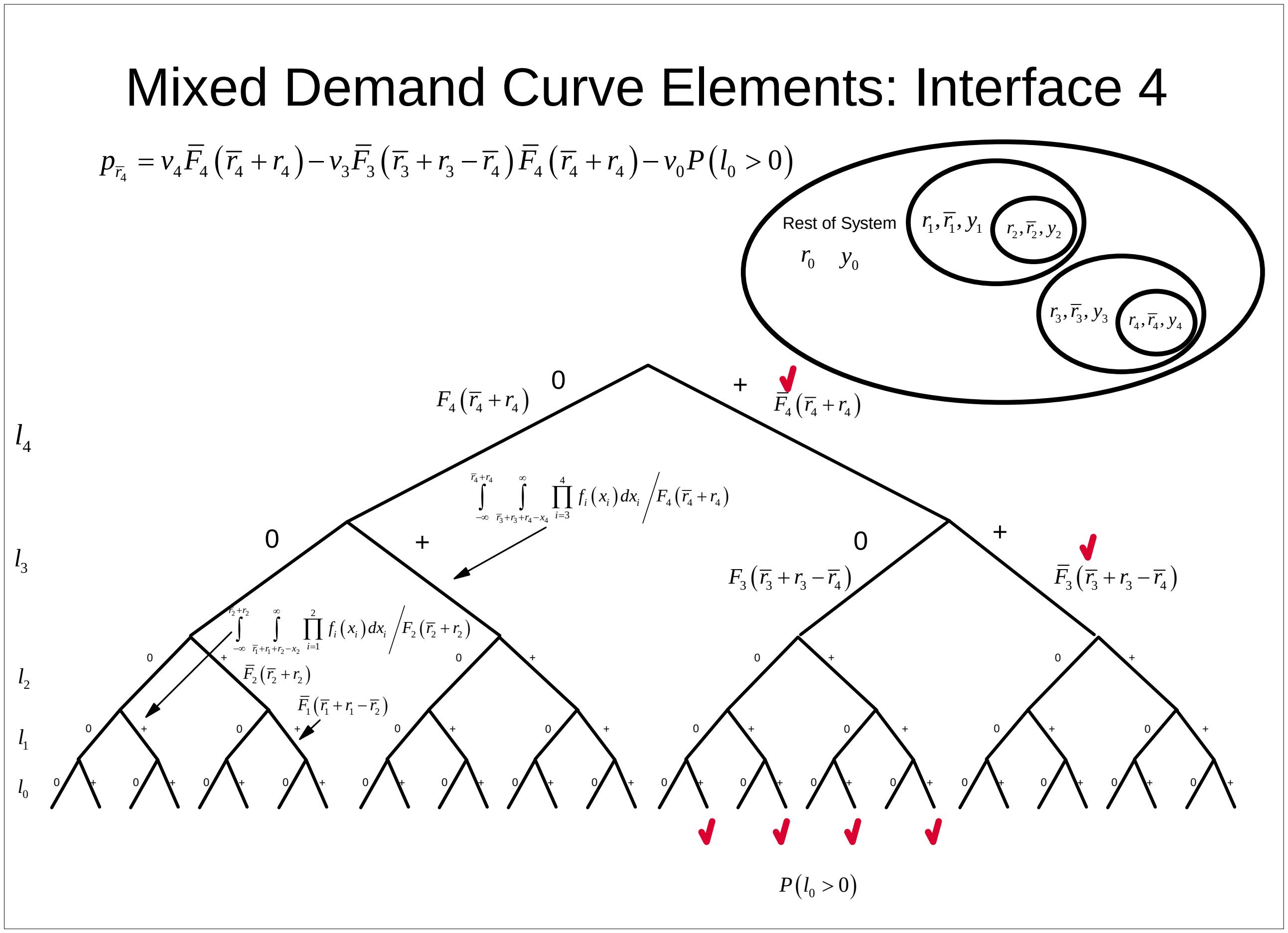
**The mixed loss outcomes and dependencies determine the demand for rest of system operating reserves.**



**The mixed loss outcomes and dependencies determine the demand for zone 4 operating reserves.**



**The mixed loss outcomes and dependencies determine the demand for interface 4 capacity.**



William W. Hogan is the Raymond Plank Professor of Global Energy Policy, John F. Kennedy School of Government, Harvard University and a Director of LECG, LLC. This paper draws on work for the Harvard Electricity Policy Group and the Harvard-Japan Project on Energy and the Environment. The author is or has been a consultant on electric market reform and transmission issues for Allegheny Electric Global Market, American Electric Power, American National Power, Australian Gas Light Company, Avista Energy, Barclays, Brazil Power Exchange Administrator (ASMAE), British National Grid Company, California Independent Energy Producers Association, California Independent System Operator, Calpine Corporation, Canadian Imperial Bank of Commerce, Centerpoint Energy, Central Maine Power Company, Chubu Electric Power Company, Citigroup, Comision Reguladora De Energia (CRE, Mexico), Commonwealth Edison Company, COMPETE Coalition, Conectiv, Constellation Power Source, Coral Power, Credit First Suisse Boston, DC Energy, Detroit Edison Company, Deutsche Bank, Duquesne Light Company, Dynegy, Edison Electric Institute, Edison Mission Energy, Electricity Corporation of New Zealand, Electric Power Supply Association, El Paso Electric, GPU Inc. (and the Supporting Companies of PJM), Exelon, GPU PowerNet Pty Ltd., GWF Energy, Independent Energy Producers Assn, ISO New England, Luz del Sur, Maine Public Advocate, Maine Public Utilities Commission, Merrill Lynch, Midwest ISO, Mirant Corporation, JP Morgan, Morgan Stanley Capital Group, National Independent Energy Producers, New England Power Company, New York Independent System Operator, New York Power Pool, New York Utilities Collaborative, Niagara Mohawk Corporation, NRG Energy, Inc., Ontario IMO, Pepco, Pinpoint Power, PJM Office of Interconnection, PPL Corporation, Public Service Electric & Gas Company, PSEG Companies, Reliant Energy, Rhode Island Public Utilities Commission, San Diego Gas & Electric Corporation, Sempra Energy, SPP, Texas Genco, Texas Utilities Co, Tokyo Electric Power Company, Toronto Dominion Bank, TransÉnergie, Transpower of New Zealand, Westbrook Power, Western Power Trading Forum, Williams Energy Group, and Wisconsin Electric Power Company. The views presented here are not necessarily attributable to any of those mentioned, and any remaining errors are solely the responsibility of the author. (Related papers can be found on the web at www.whogan.com ).