SCARCITY PRICING: MORE ON LOCATIONAL OPERATING RESERVE DEMAND CURVES

William W. Hogan

Mossavar-Rahmani Center for Business and Government
John F. Kennedy School of Government
Harvard University
Cambridge, Massachusetts 02138

Harvard Electricity Policy Group

San Diego, CA

March 13, 2009 (Revised March 23, 2009)

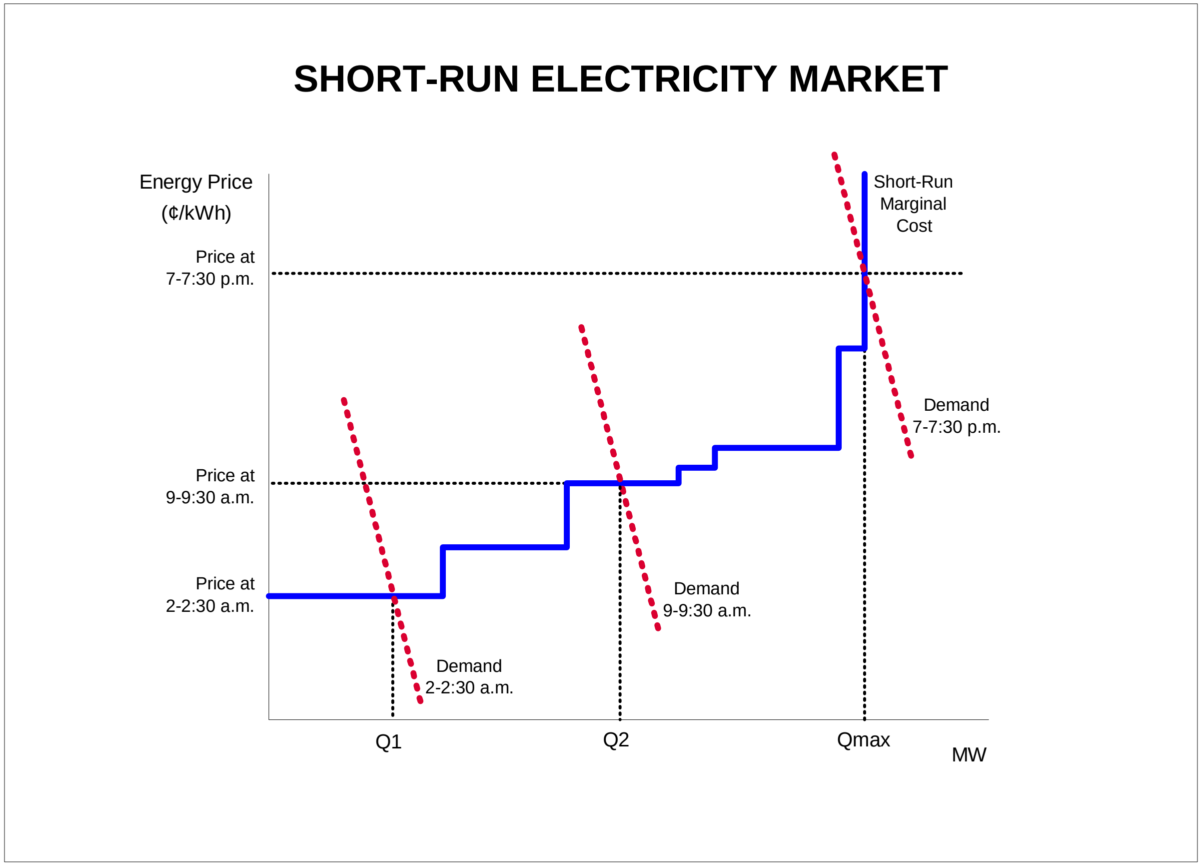
Scarcity pricing presents one of the important challenges for Regional Transmission Organizations (RTOs) and electricity market design. Simple in principle, but more complicated in practice, inadequate scarcity pricing is implicated in several problems associated with electricity markets.

- **Investment Incentives.** Inadequate scarcity pricing contributes to the "missing money" needed to support new generation investment. The policy response has been to create capacity markets. Better scarcity pricing would reduce the challenges of operating good capacity markets.
- **Demand Response.** Higher prices during critical periods would facilitate demand response and distributed generation when it is most needed. The practice of socializing payments for capacity investments compromises the incentives for demand response and distributed generation.
- **Renewable Energy.** Intermittent energy sources such as solar and wind present complications in providing a level playing field in pricing. Better scarcity pricing would reduce the size and importance of capacity payments and improve incentives for renewable energy.
- **Transmission Pricing.** Scarcity pricing interacts with transmission congestion. Better scarcity pricing would provide better signals for transmission investment.

Improved scarcity pricing would mitigate or substantially remove the problems in all these areas. While long-recognized, only recently has there been renewed interest in developing a better theory and practice of scarcity pricing.[1]

1 FERC, Order 719, October 17, 2008.

Early market designs presumed a significant demand response. Absent this demand participation most markets implemented inadequate pricing rules equating prices to marginal costs even when capacity is constrained. This produces a "missing money" problem.



The theory and practice of scarcity pricing intersect important elements of electricity systems and economic dispatch.

- **Reliability.** By definition, scarcity conditions arise when the system is constrained and dispatch is modified to respect reliability constraints.
- **Dispatch.** Simultaneous optimization of energy and reserves means that scarcity in either effects prices for both.
- **Resource Adequacy.** The standards for resource adequacy and operating security are not fully integrated or compatible.

A critical connection is the treatment of operating reserves and construction of operating reserve demand curves. The basic idea of applying operating reserve demand curves is well tested and found in operation in important RTOs.

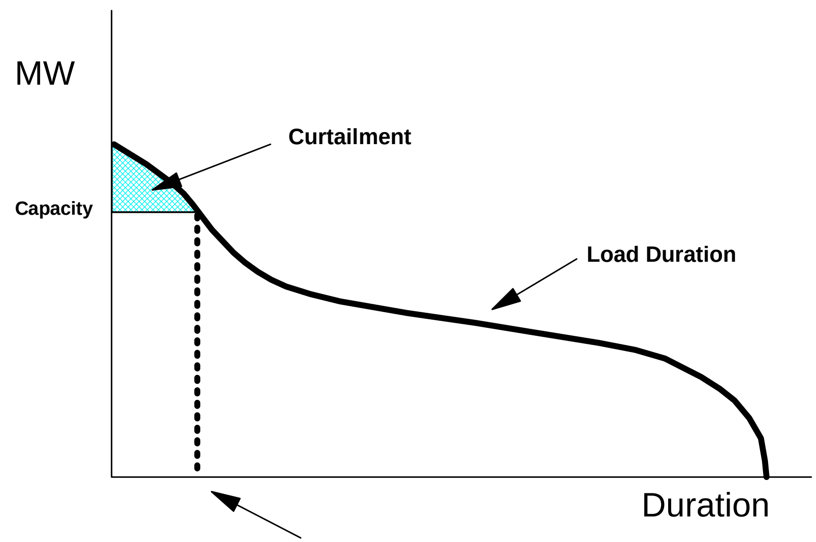
- **NYISO.** See NYISO Ancillary Service Manual, Volume 3.11, Draft, April 14, 2008, pp, 6-19-6-22.
- **ISONE.** FERC Electric Tariff No. 3, Market Rule I, Section III.2.7, February 6, 2006.
- **MISO.** FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009.

The underlying models of operating reserve demand curves differ across RTOs. One need is for a framework that develops operating reserve demand curves from first principles to provide a benchmark for the comparison of different implementations.

- **Operating Reserve Demand Curve Components.** The inputs to the operating reserve demand curve construction can differ and a more general model would help specify the result.
- **Locational Differences and Interactions.** The design of locational operating reserve demand curves presents added complications in accounting for transmission constraints.
- **Economic Dispatch.** The derivation of the locational operating demand curves has implications for the integration with economic dispatch models for simultaneous optimization of energy and reserves.

There is a simple stylized connection between reliability standards and resource economics. Defining expected load shedding duration, choosing installed capacity, or estimating value of lost load address different facets of the same problem.

A Simple Reliability Model

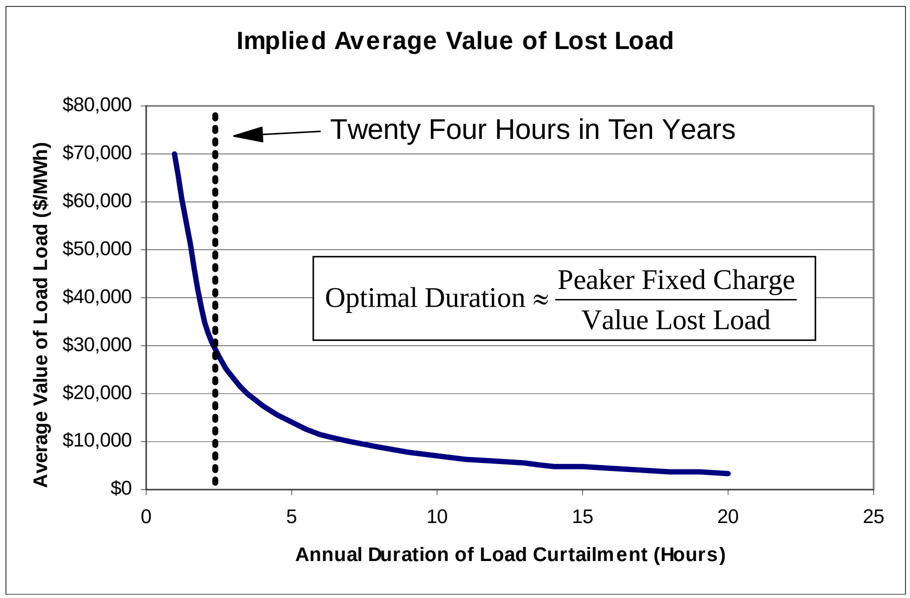


$$\text{Optimal Duration} \approx \frac{\text{Peaker Fixed Charge}}{\text{Value Lost Load}}$$

(Steven Stoft, Power System Economics, IEE Press, Wiley Interscience, 2002, p. 138)

The simple connection between reliability planning standards and resource economics illustrates a major disconnect between market pricing and the implied value of lost load.

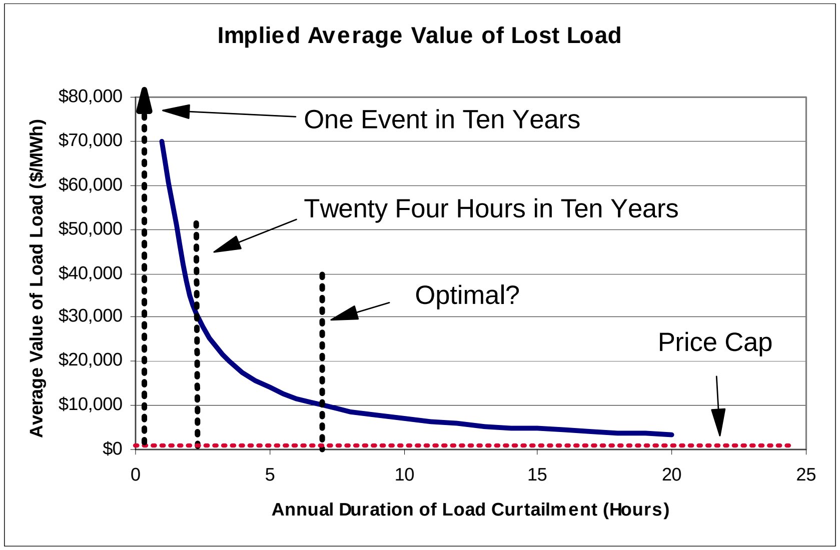
Reliability Planning Standard and Value of Lost Load



Peaker fixed charge at $65,000/MW-yr.

There is a large disconnect between long-term planning standards and market design. The installed capacity market analyses illustrate the gap between prices and implied values. The larger disconnect is between the operating reserve market design and the implied reliability standard.

Reliability Standard and Market Disconnect



Peaker fixed charge at $65,000/MW-yr.

Implied prices differ by orders of magnitude. $\left(\text{Price Cap} \approx \$10^3;\ \text{VOLL} \approx \$10^4;\ \text{Reliability Standard} \approx \$10^5\right)$

Locational fixed operating reserve minimums are already familiar practice. The detailed operating rules during reserve scarcity involve many steps. Improved scarcity pricing accompanies an operating reserve demand curve under dispatch-based pricing. Consider a simplified setting.

- **Dispatched-Based Pricing.** Interpret the actual dispatch result as the solution of the reliable economic dispatch problem. Calculate consistent prices from the simplified model.
- **Single Period.** Unit commitment decisions made as though just before the start of the period. Uncertain outcomes determined after the commitment decision, with only redispatch or emergency actions such as curtailment over the short operating period (e.g. less than an hour).
- **Single Reserve Class.** Model operating reserves as committed and synchronized.
- **DC Network Approximation.** Focus on role of reserves but set context of simultaneous dispatch of energy and reserves. A network model for energy, but a zonal model for reserves.

The purpose here is to pursue further development of the properties of a market model that expands locational reserve requirements to include operating reserve demand curve(s). The NYISO, ISONE, MISO market designs include locational operating reserve demand curves.

As outlined in an appendix, an expected value formulation of economic dispatch reduces to a much more manageable scale with the introduction of the implicit value of expected unserved energy (VEUE) function.

$$\underset{y^0, d^0, g^0, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right)$$

$$s.t.$$

$$y^0 = d^0 - g^0,$$

$$H^0 y^0 \le b^0,$$

$$g^0 + r \le u \bullet Cap^0,$$

$$\iota^t y^0 = 0,$$

$$g^0 \le u \bullet Cap^0.$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

Ignore the network features for the first illustration. Assume all the load and generations is at a single location. Focus on the deviations form the base dispatch. Unserved energy demand is a random variable with a distribution for the probability that load exceeds available capacity.

$$Unserved\ Energy = Max\left(0, Load - Available\ Capacity\right)$$

Hence

$$\begin{aligned} &Unserved\ Energy = Max\Big(0, E\left(Load\right) + \Delta\ Load - \left(Committed\ Capacity - \Delta Capacity\right)\Big) \\ &= Max\Big(0, \Delta\ Load + Outage + \big(E\left(Load\right) - Committed\ Capacity\big)\Big) \\ &= Max\Big(0, \Delta\ Load + Outage - Operating\ Reserve\Big). \end{aligned}$$

This produces the familiar loss of load probability (*LOLP*) calculation, for which there is a long history of analysis and many techniques. With operating reserves (*r*),

$$LOLP = \Pr\left(\Delta\ Load + Outage \geq r\right) = \overline{F}_{LOL}\left(r\right).$$

A common characterization of a reliability constraint is that there is a limit on the *LOLP.* This imposes a constraint on the required reserves (*r*).

$$\overline{F}_{LOL}\left(r\right) \leq LOLP_{Max}.$$

This constraint formulation implies an infinite cost for unserved energy above the constraint limit, and zero value for unserved energy that results within the constraint.

An alternative approach is to consider the expected unserved energy (*EUE*) and the Value of Lost Load (*VOLL*).

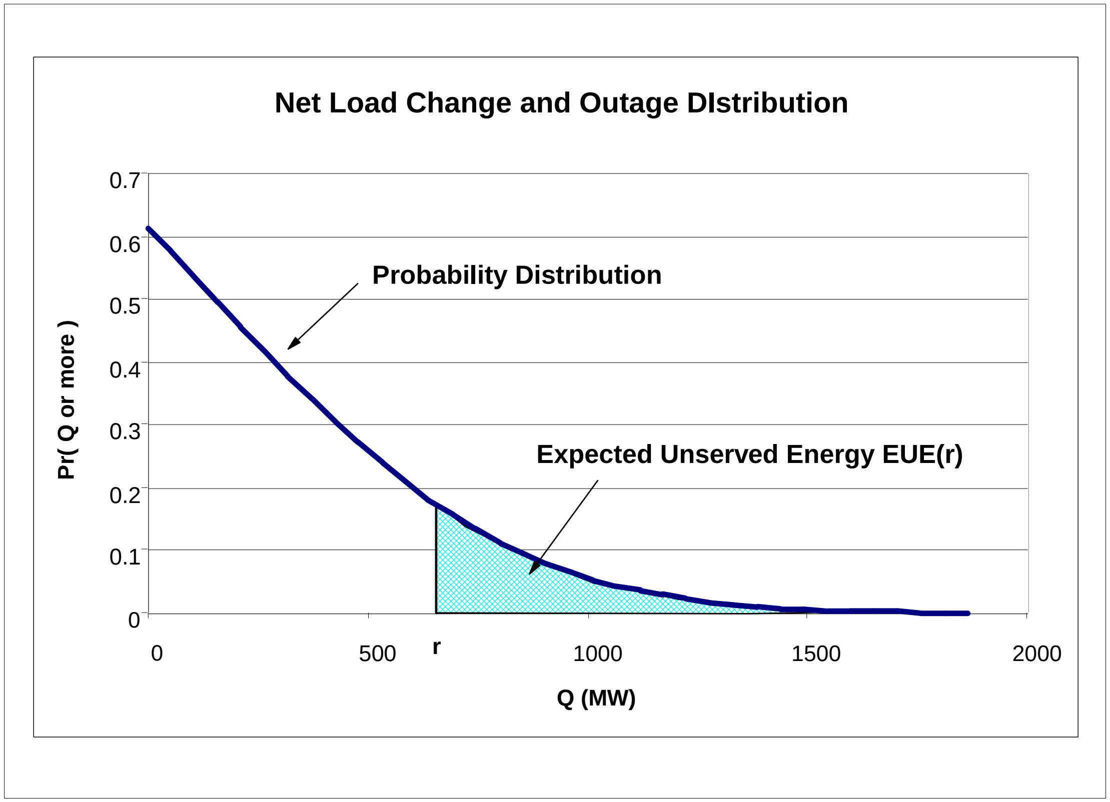
Suppose the *VOLL* per MWh is v. Then we can obtain the *EUE* and its total value (*VEUE*) as:

$$EUE(r) = \int_r^{\infty} \overline{F}_{LOL}(x)\,dx.$$

$$VEUE(r) = v\int_r^{\infty} \overline{F}_{LOL}(x)\,dx.$$

There is a chance that no outage occurs and that net load is less than expected, or $\overline{F}_{LOL}(0) < 1.$

The real changes may not be continuous, but it is common to apply continuous approximations. Total value of expected unserved energy is of same magnitude as the cost of meeting load.



The distribution of load and facility outages compared to operating reserves determines the LOLP.

A reasonable approximation is that the change in load is normally distributed: $\Delta\, Load \sim N\left(0, \sigma_L^2\right)$.

The outage distribution is more complicated and depends on many factors, including the unit commitment. Suppose that $o_j = 0,1$ is a random variable where $o_j = 1$ represents a unit outage. The probability of an outage in the monitored period, given that plant was available and committed at the start of the period ($u_j = 1$) is ω_j, typically a small value on the order of less than 10^{-2}:

$$Outage = \sum_j u_j Cap_j o_j,$$

$$\Pr\left(o_j = 1 \middle| u_j = 1\right) = \omega_j.$$

A common approximation of $\Pr(Outage)$ is a mixture of distributions with a positive probability of no outage and a conditional distribution of outages that follows an exponential distribution.[2]

$$\Pr\left(Outage = 0\right) = p_0, \Pr\left(Outage > x\right) = \left(1 - p_0\right) e^{-\lambda x}.$$

The combined distribution for change in load and outages can be complicated.[3] In application, this distribution might be estimated numerically, possibly from Monte Carlo simulations.

---

[2] Debabrata Chattopadhyay and Ross Baldick, "Unit Commitment with Probabilistic Reserve," IEEE, Power Engineering Society Winter Meeting, Vol. 1, pp. 280-285.

[3] Guy C. Davis, Jr., and Michael H. Kutner, "The Lagged Normal Family Of Probability Density Functions Applied To Indicator-Dilution Curves," Biometrics, Vol. 32, No. 3, September 1976, pp. 669-675.

For sake of the present illustration, make a simplifying assumption that the outage distribution is approximated by a normal distribution.

$$Outage \sim N\left(\mu_O, \sigma_O^2\right).$$

Then with operating reserves *r*, the distribution of the lost load is

$$LOLP = \Pr\left(\Delta\ Load + Outage \geq r\right) = \bar{F}_{LOL}\left(r\right)$$
$$= \bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) = 1 - \Phi\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

Here $\Phi\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right)$ is the cumulative normal distribution with mean and variance $\mu_O, \sigma_O^2 + \sigma_L^2$.

$$EUE\left(r\right) = \int_r^\infty \bar{\Phi}\left(x \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) dx.$$

$$VEUE\left(r\right) = v \int_r^\infty \bar{\Phi}\left(x \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right) dx.$$

This gives the implied reserve inverse demand curve as

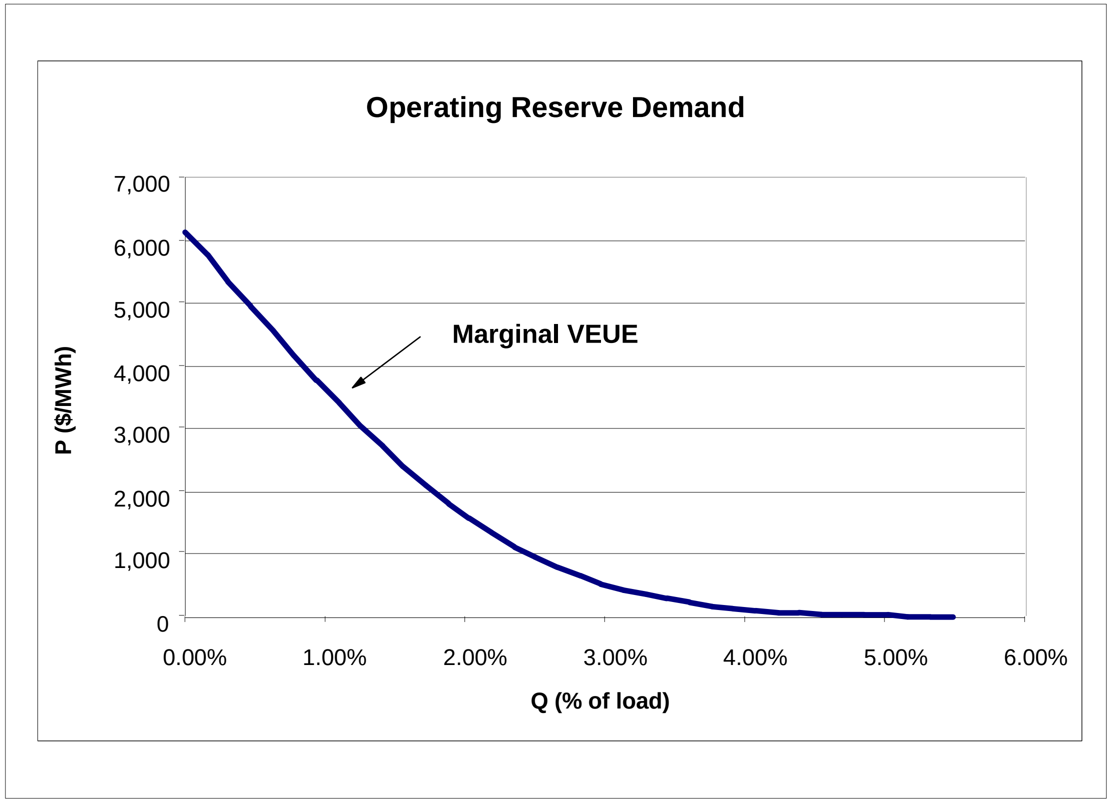
$$Operating\ Reserve\ Demand\ Price\left(r\right) = P_{OR}\left(r\right) = v\bar{\Phi}\left(r \middle| \mu_O, \sigma_O^2 + \sigma_L^2\right).$$

Operating reserve demand is a complement to energy demand for electricity. The probabilistic demand for operating reserves reflects the cost and probability of lost load.[4]

Example Assumptions

Expected Load (MW)	34000
Std Dev %	1.50%
Expected Outage %	0.45%
Std Dev %	0.45%
Expected Total (MW)	153
Std Dev (MW)	532.46
VOLL ($/MWh)	10000

Under the simplifying assumptions, if the dispersion of the LOLP distribution is proportional to the expected load, the operating reserve demand is proportional to the expected load.



[4] "For each cleared Operating Reserve level less than the Market-Wide Operating Reserve Requirement, the Market-Wide Operating Reserve Demand Curve price shall be equal to the product of (i) the Value of Lost Load ("VOLL") and (ii) the estimated conditional probability of a loss of load given that a single forced Resource outage of 100 MW or greater will occur at the cleared Market-Wide Operating Reserve level for which the price is being determined. … The VOLL shall be equal to $3,500 per MWh." MISO, FERC Electric Tariff, Volume No. 1, Schedule 28, January 22, 2009, Sheet 2226.

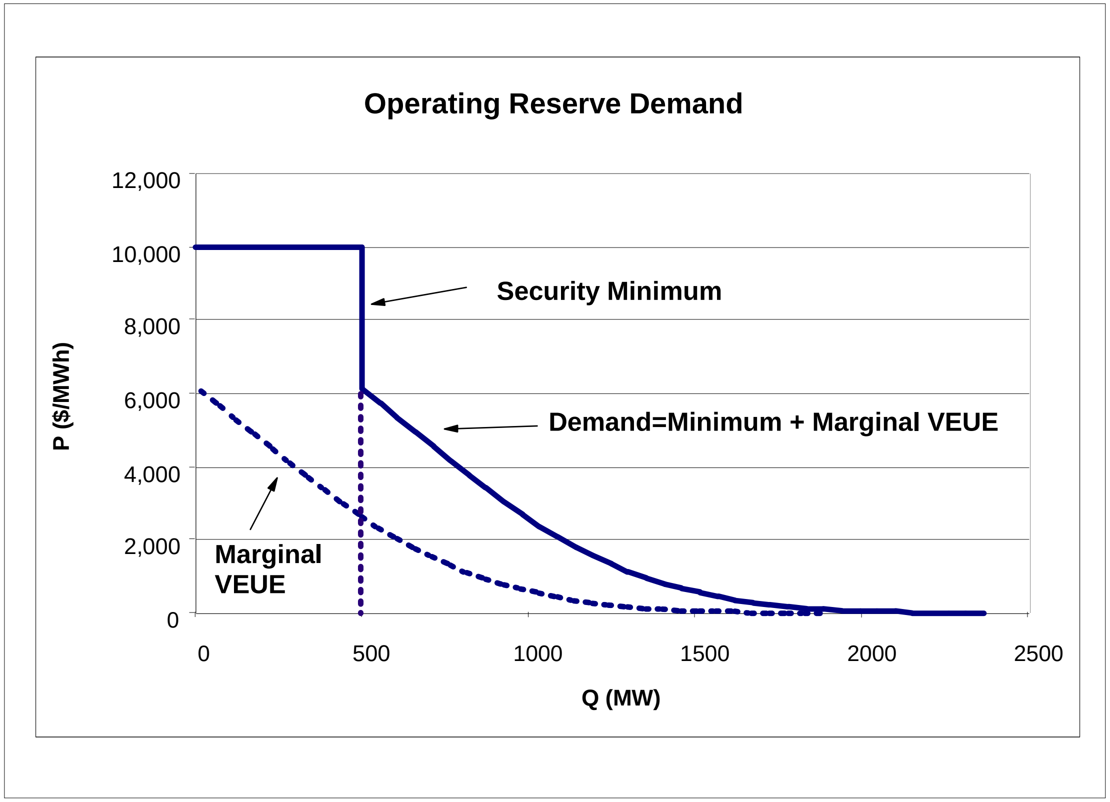
The deterministic approach to security constrained economic dispatch includes lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve to maintain the system for an emergency period.

Suppose that the maximum generation outage contingency quantity is $r_{Min}\left(d^0, g^0, u\right)$. Then we would have the constraint:

$$r \geq r_{Min}\left(d^0, g^0, u\right).$$

In effect, the contingency constraint provides a vertical demand curve that adds horizontally to the probabilistic operating reserve demand curve.

If the security minimum will always be maintained over the monitored period, the VEUE price at r=0 applies. If the outage shocks allow excursions below the security minimum during the period, the VEUE starts at the security minimum.



In a network, security constrained economic dispatch includes a set of monitored transmission contingencies, K_M , with the transmission constraints on the pre-contingency flow determined by conditions that arise in the contingency.

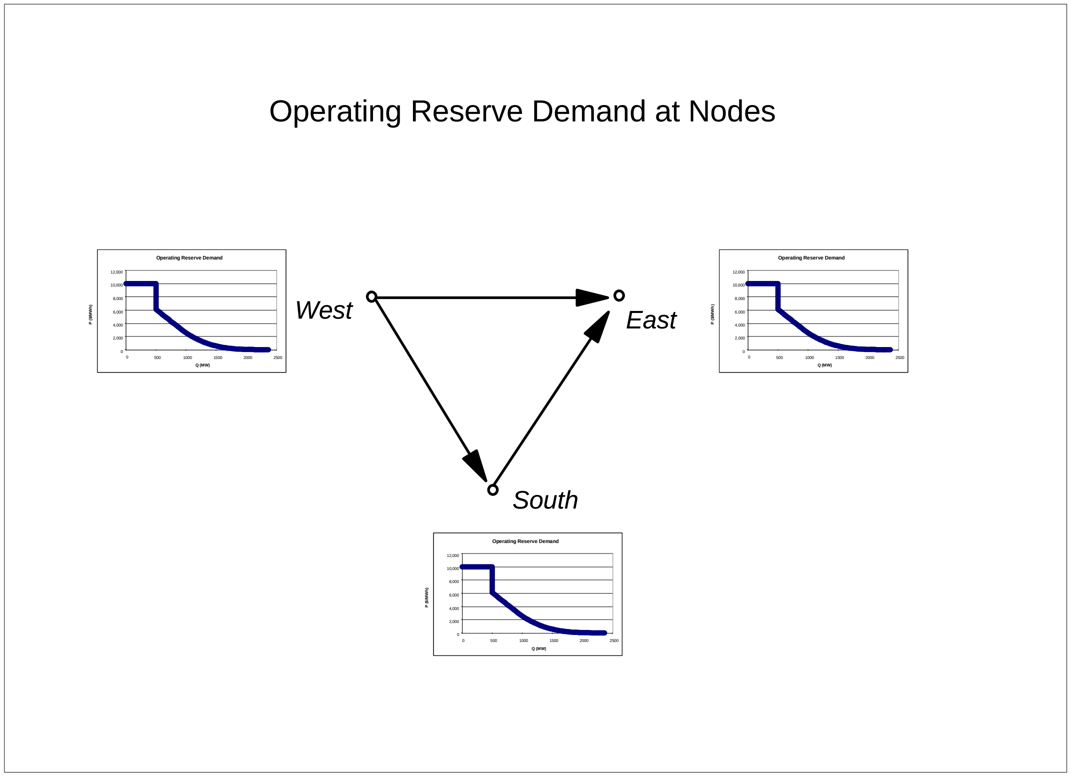
$$H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M .$$

The security constrained economic dispatch problem becomes:

$$\begin{aligned}
&\underset{y^0, d^0, g^0, r, u \in (0,1)}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - VEUE\left(d^0, g^0, r, u\right) \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad H^0 y^0 \le b^0, \\
&\quad H^i y^0 \le \tilde{b}^i, \quad i = 1, 2, \cdots, K_M, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad r \ge r_{Min}\left(d^0, g^0, u\right) \\
&\quad \iota^t y^0 = 0, \\
&\quad g^0 \le u \bullet Cap^0.
\end{aligned}$$

If we could convert each node to look like the single location examined above, the approximation of *VEUE*, would repeat the operating reserve demand curve at each node.

Conceptually we could think of the *LOLP* distribution at each location.[5] This would give rise to an operating reserve demand curve at each location.



[5] Eugene G. Preston, W. Mack Grady, Martin L. Baughman, "A New Planning Model for Assessing the Effects of Transmission Capacity Constraints on the Reliability of Generation Supply for Large Nonequivalenced Electric Networks," IEEE Transactions on Power Systems, Vol. 12, No. 3, August 1997, pp. 1367-1373. J. Choi, R. Billinton, and M. Futuhi-Firuzabed, "Development of a Nodal Effective Load Model Considering Transmission System Element Unavailabilities," IEE Proceedings - Generation, Transmission and Distribution, Vol. 152, No. 1, January 2005, pp. 79-89.

A difficulty with defining a locational operating reserve demand curve is the complexity of the interactions among locations plus interactions with the transmission grid. A similar problem appears in the evaluation of planned transmission and generation investment.

- **Expected Values.** The basic formulation of the real-time economic dispatch problem is built on a particular configuration of the transmission grid and the usual application of Kirchoff's laws. The operating reserve and long-term planning problem share a focus on the expected values of outcomes across different conditions. The expected value in principle applies probabilities across many configurations and the expected value need not follow the individual dictates of Kirchoff's laws.

- **Zonal Model.** The expected value formulation rationalizes approximation in a zonal model. The zonal application across a wide range of conditions is a regular feature of RTO transmission planning and resource adequacy calculations.

  - **Zones with Closed Interfaces.** Areas with limited transmission are defined and treated as having a close interface with a capacity limit for emergency transfers from the rest of the system.

  - **Capacity Emergency Transfer Limit (CETL).** Conservative transmission standards (e.g., 1 day in 25 years) apply to simulations that determine the transfer limit.[6]

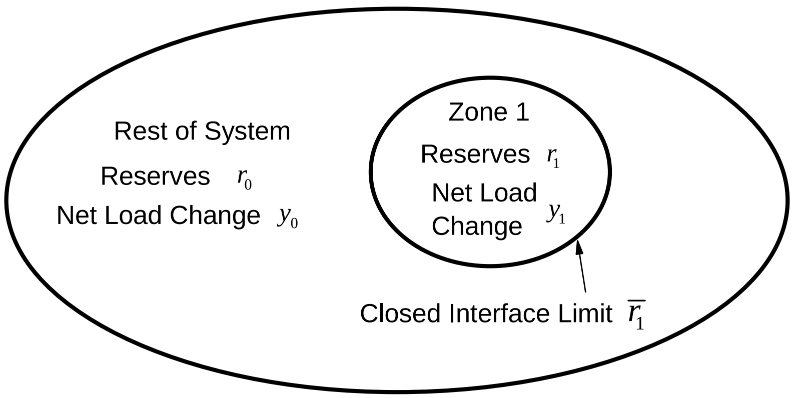
- **Interface Limits.** Although the exact CETL calculations might not be appropriate for short-term reserve management, the analogy could be applied to determine closed interface limits.

---

6 PJM , *2008 PJM Reserve Requirement Study*, October 8, 2008, Appendix H.

The task is to define a locational operating reserve model that approximates and prices the dispatch decisions made by operators. To illustrate, consider the simplest case with one constrained zone and the rest of the system. The reserves are defined separately and there is a known transfer limit for the closed interface between the constrained zone and the rest of the system.

Zonal Interface Limit on Emergency Transfers

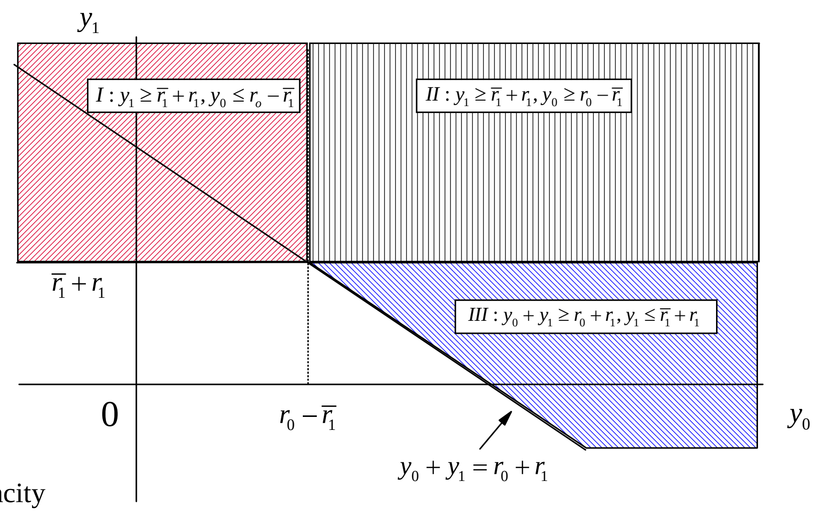


$$y_0 \sim f_0, y_1 \sim f_1, F_o(y_0) = \int_{-\infty}^{y_0} f_0(x_0)\,dx_0, F_1(y_1) = \int_{-\infty}^{y_1} f_1(x_1)\,dx_1$$

The basic emergency dispatch problem is to determine the configuration of lost load. And the expected value of the loss load defines the zonal value of expected unserved energy.

$$ZVEUE\left(r_0,\overline{r}_1,r_1\right)=E_y\left[\underset{l_i\geq 0}{Min}\quad\left\{v_0 l_0+v_1 l_1 \middle| y_0+y_1-l_0-l_1\leq r_0+r_1,\, y_1-l_1\leq\overline{r}_1+r_1\right\}\right]$$

Model for Zonal Operating Reserve and Load Loss



$\overline{r}_1=$ Interface Reserve Capacity

$r_0=$ Reserves Outside Zone

$r_1=$ Reserves Inside Zone

$y_0=$ Net Load Change Outside Zone

$y_1=$ Net Load Change Inside Zone

$I:$ load loss $l_1, VOLL_1=v_1$

$II:$ load loss $l_1, VOLL_1=v_1$, load loss $l_0, VOLL_0=v_0$

$III:$ load loss $l_0, VOLL_0=v_0$

$v_1\geq v_0$

The basic emergency dispatch problem is to determine the configuration of lost load. Examination of the possible configurations of outages reveals the marginal values of the zonal value of unserved energy, which define the locational demand curves for operating reserves.

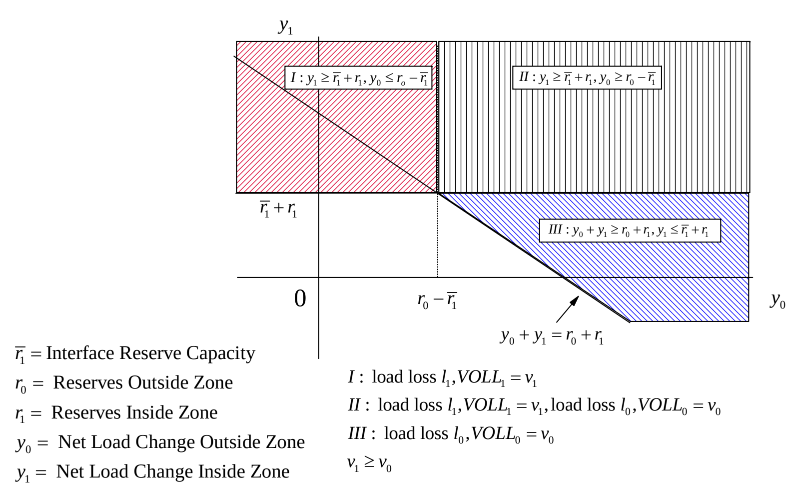
$$p_{r_1} = -\frac{\partial ZVEUE\left(r_0, \overline{r}_1, r_1\right)}{\partial r_1} = v_1 P\left(I + II\right) + v_0 P\left(III\right) = v_1 P\left(y_1 \geq \overline{r}_1 + r_1\right) + v_0 P\left(y_0 + y_1 \geq r_0 + r_1, y_1 \leq \overline{r}_1 + r_1\right)$$

$$p_{\overline{r}_1} = -\frac{\partial ZVEUE\left(r_0, \overline{r}_1, r_1\right)}{\partial \overline{r}_1} = v_1 P\left(I + II\right) - v_0 P\left(II\right) = v_1 P\left(y_1 \geq \overline{r}_1 + r_1\right) - v_0 P\left(y_0 \geq r_0 - \overline{r}_1, y_1 \geq \overline{r}_1 + r_1\right)$$

$$p_{r_0} = -\frac{\partial ZVEUE\left(r_0, \overline{r}_1, r_1\right)}{\partial r_0} = v_0 P\left(II + III\right) = v_0 \left[P\left(y_0 + y_1 \geq r_0 + r_1, y_1 \leq \overline{r}_1 + r_1\right) + P\left(y_0 \geq r_0 - \overline{r}_1, y_1 \geq \overline{r}_1 + r_1\right)\right]$$

$$= v_0 P\left(y_0 + y_1 \geq r_0 + r_1, y_0 \geq r_0 - \overline{r}_1\right)$$

Model for Zonal Operating Reserve and Load Loss



As explained in an appendix, these operating reserve demand curves can be reduced to probability calculations in terms of the distributions of net load changes in the constrained zone and the rest of the system.

$$
\begin{aligned}
p_{r_1} &= v_1\left(1-F_1\left(\overline{r}_1+r_1\right)\right)+v_0\int_{-\infty}^{\overline{r}_1+r_1}\left[1-F_0\left(r_0+r_1-x_1\right)\right] f_1\left(x_1\right)dx_1 \\
p_{\overline{r}_1} &= v_1\left(1-F_1\left(\overline{r}_1+r_1\right)\right)-v_0\left(1-F_0\left(r_0-\overline{r}_1\right)\right)\left(1-F_1\left(\overline{r}_1+r_1\right)\right) \\
p_{r_0} &= v_0\int_{r_0-\overline{r}_1}^{\infty}\left[1-F_1\left(r_0+r_1-x_0\right)\right] f_0\left(x_0\right)dx_0
\end{aligned}
$$

The implied demand curves illustrate critical properties.

- **Interaction.** The demand curves are interdependent, but the dependence is not in the form of the nested or cascading model often assumed.
- **Convergence.** As the interface capacity increases, the implied demand curves in the constrained zone and for the rest of the system converge to the same prices.
- **Interface Demand.** In addition to the demand for operating reserves, there is an implied demand curve for the interface transfer limit.
- **No Thresholds.** The implied demand curve scarcity prices are positive at all levels. At higher reserves the prices are lower, but there is no threshold where the scarcity price falls to zero.

Using the same example as above, we separate the system into two zones with independent probability distributions. The expected total outage, standard deviations, and VOLLs are consistent with the unconstrained example above.

	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

Virtually any realistic distributional could be accommodated. For the sake of the illustration, continue the assumption that the individual net load distributions follow a normal approximation.

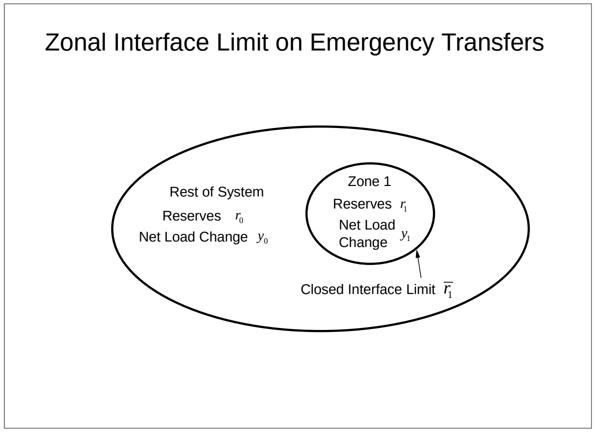
The resulting demand curves all depend on all the parameters. For a given benchmark of the values for operating reserves and the interface constraint, we can calculate the associated prices and trace out the implied demand curves when varying one dimension while holding the others constant. For the examples that follow, the benchmark reserves and interface point is:

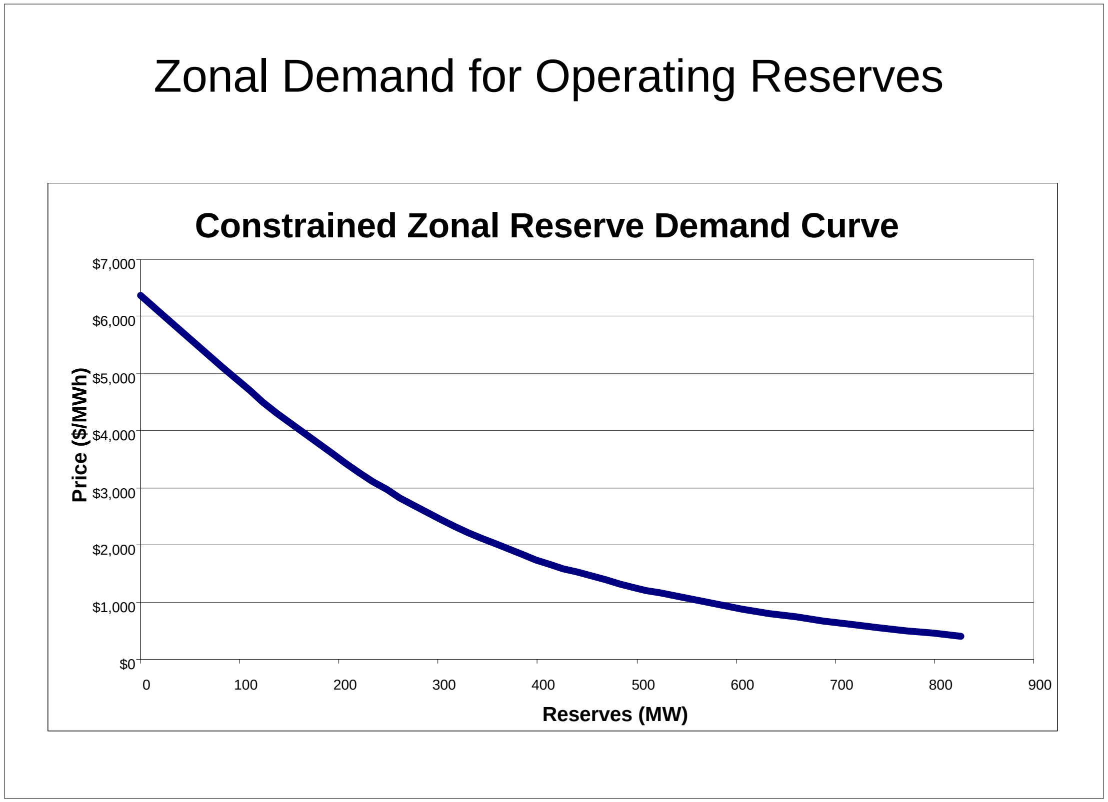
	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85

An illustrative demand curve for the constrained zone.

	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85



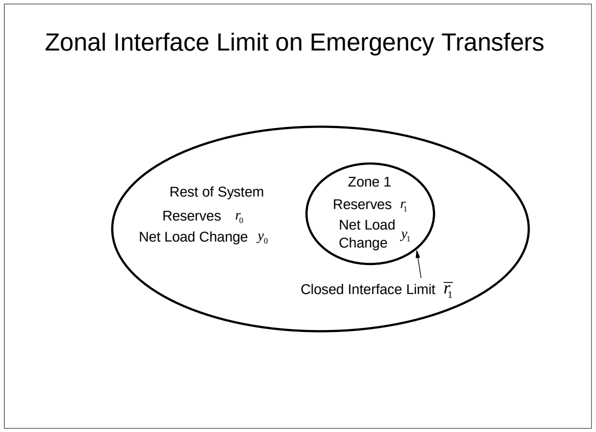


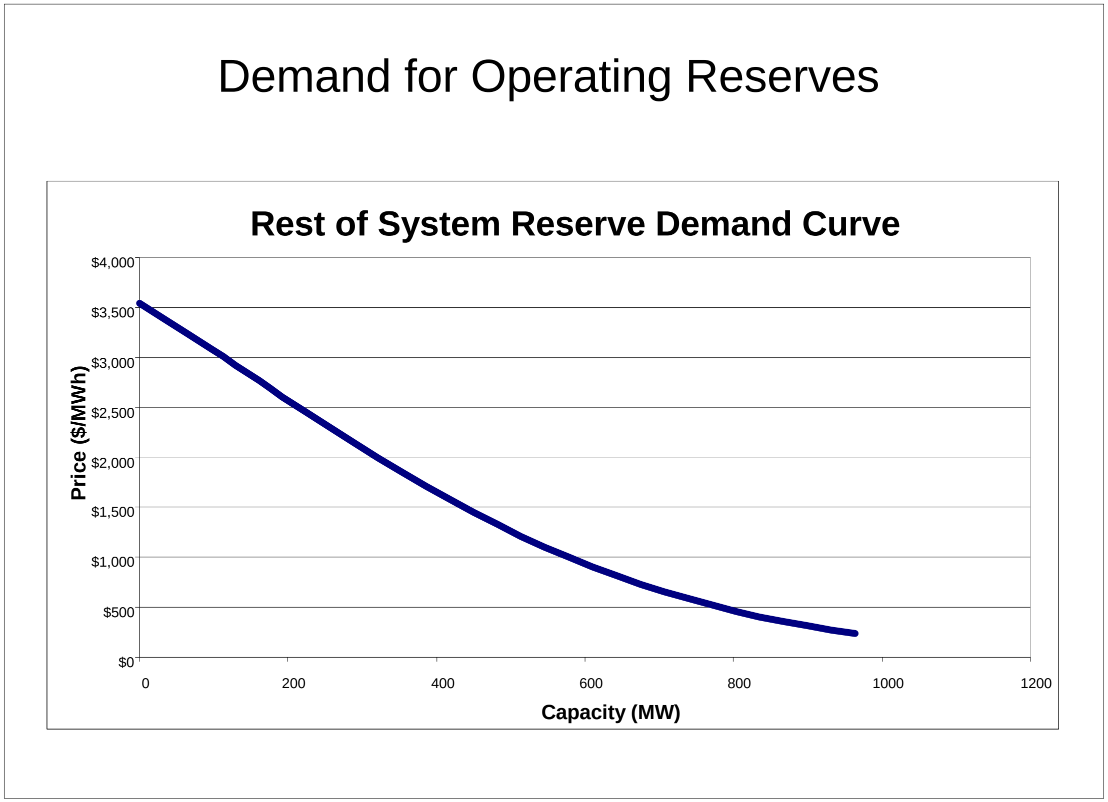
$$p_{r_1} = v_1\left(1 - F_1\left(\bar{r}_1 + r_1\right)\right) + v_0 \int_{-\infty}^{\bar{r}_1 + r_1} \left[1 - F_0\left(r_0 + r_1 - x_1\right)\right] f_1\left(x_1\right) dx_1$$

An illustrative demand curve for the rest of the system.

	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85



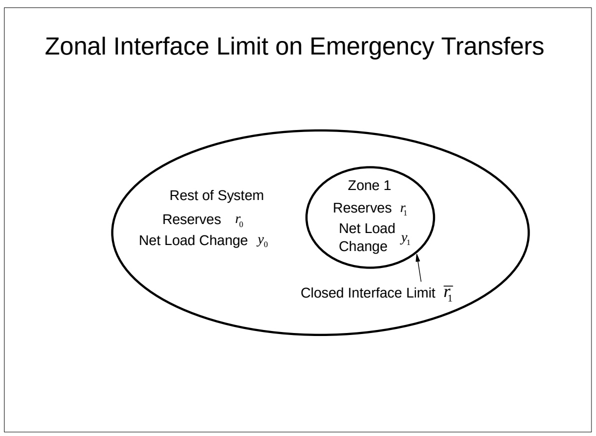


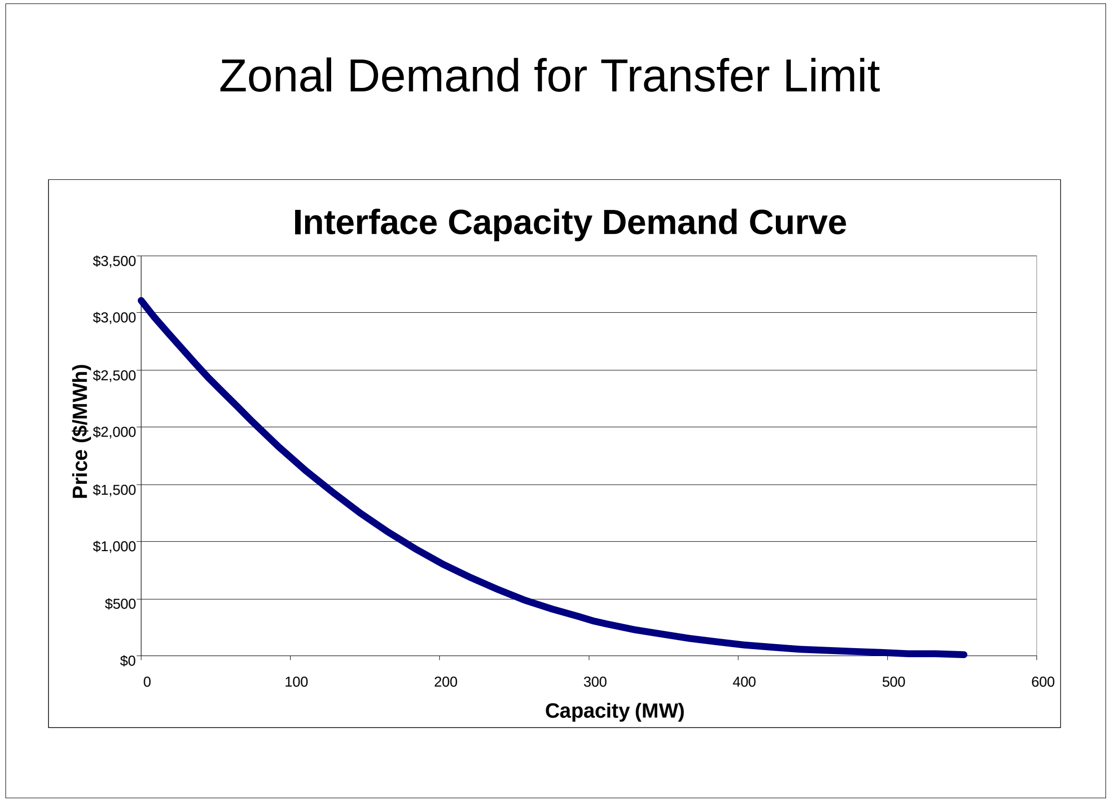
$$p_{r_0} = v_0 \int_{r_0-\overline{r}_1}^{\infty} \left[1 - F_1 \left(r_0 + r_1 - x_0 \right) \right] f_0 \left(x_0 \right) dx_0$$

An illustrative demand curve for the interface capacity.

	ROS	Zone 1
Expected Total (MW)	107.10	45.90
Std Dev (MW)	488.99	209.57
VOLL ($/MWh)	7000	10000

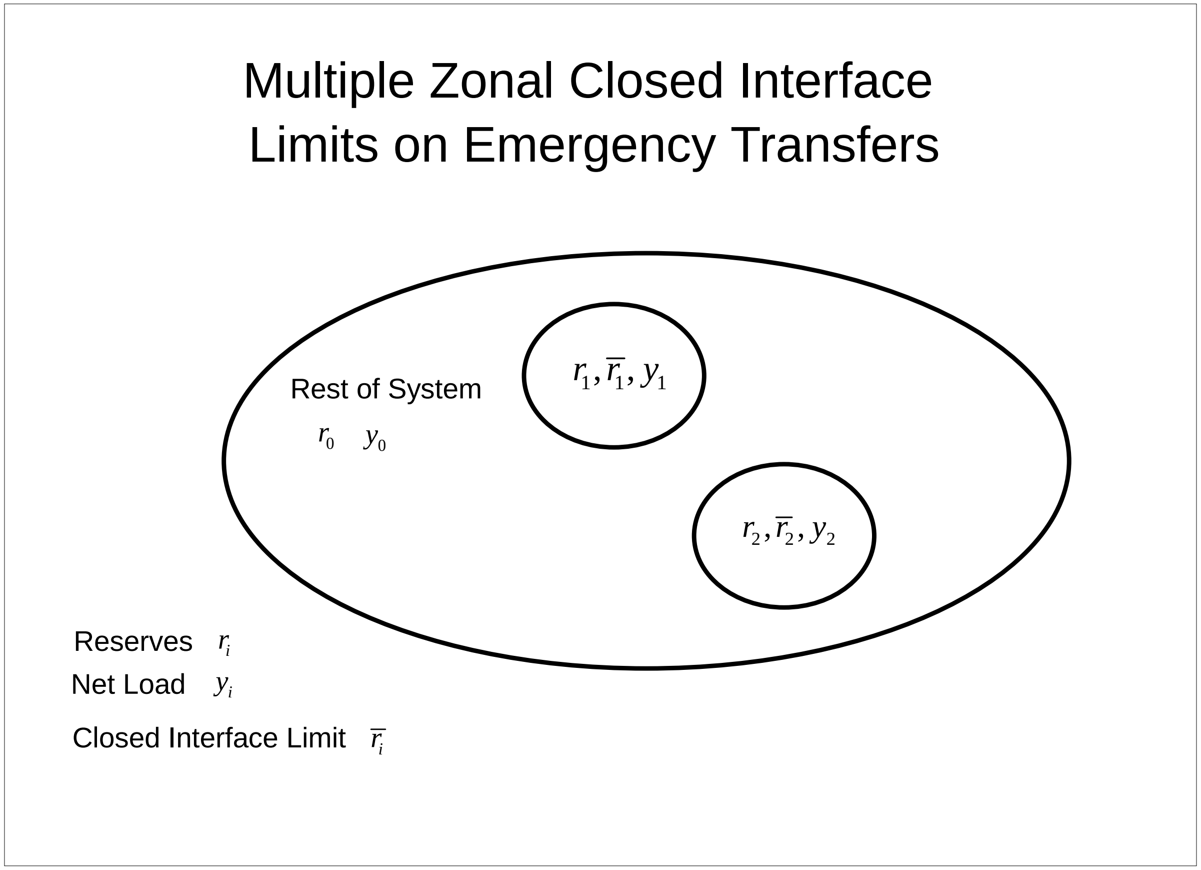
	ROS	Zone 1	Interface
Benchmark (MW)	160.65	45.90	68.85





$$p_{\overline{r}_1} = v_1\left(1-F_1\left(\overline{r}_1+r_1\right)\right)-v_0\left(1-F_0\left(r_0-\overline{r}_1\right)\right)\left(1-F_1\left(\overline{r}_1+r_1\right)\right)$$

The case of multiple constrained zones is a natural extension of the case for a single constrained zone.



With two mutually exclusive constrained zones, the possible configurations of the loss of load identify the probability model that defines the operating reserve demand curves.

$$p_{r_1} = -\frac{\partial VEUE\left(r_0,\overline{r}_1,r_1,\overline{r}_2,r_2\right)}{\partial r_1} = v_1 P\left(y_1 \geq \overline{r}_1 + r_1\right) + v_0 \begin{bmatrix} P\left(y_0+y_1+y_2 \geq r_0+r_1+r_2, y_1 \leq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \\ +P\left(y_0+y_1 \geq r_0+r_1, y_1 \leq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \end{bmatrix}$$

$$p_{r_2} = -\frac{\partial ZVEUE\left(r_0,\overline{r}_1,r_1,\overline{r}_2,r_2\right)}{\partial r_2} = v_2 P\left(y_2 \geq \overline{r}_2 + r_2\right) + v_0 \begin{bmatrix} P\left(y_0+y_1+y_2 \geq r_0+r_1+r_2, y_1 \leq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \\ +P\left(y_0+y_2 \geq r_0+r_2, y_1 \geq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \end{bmatrix}$$

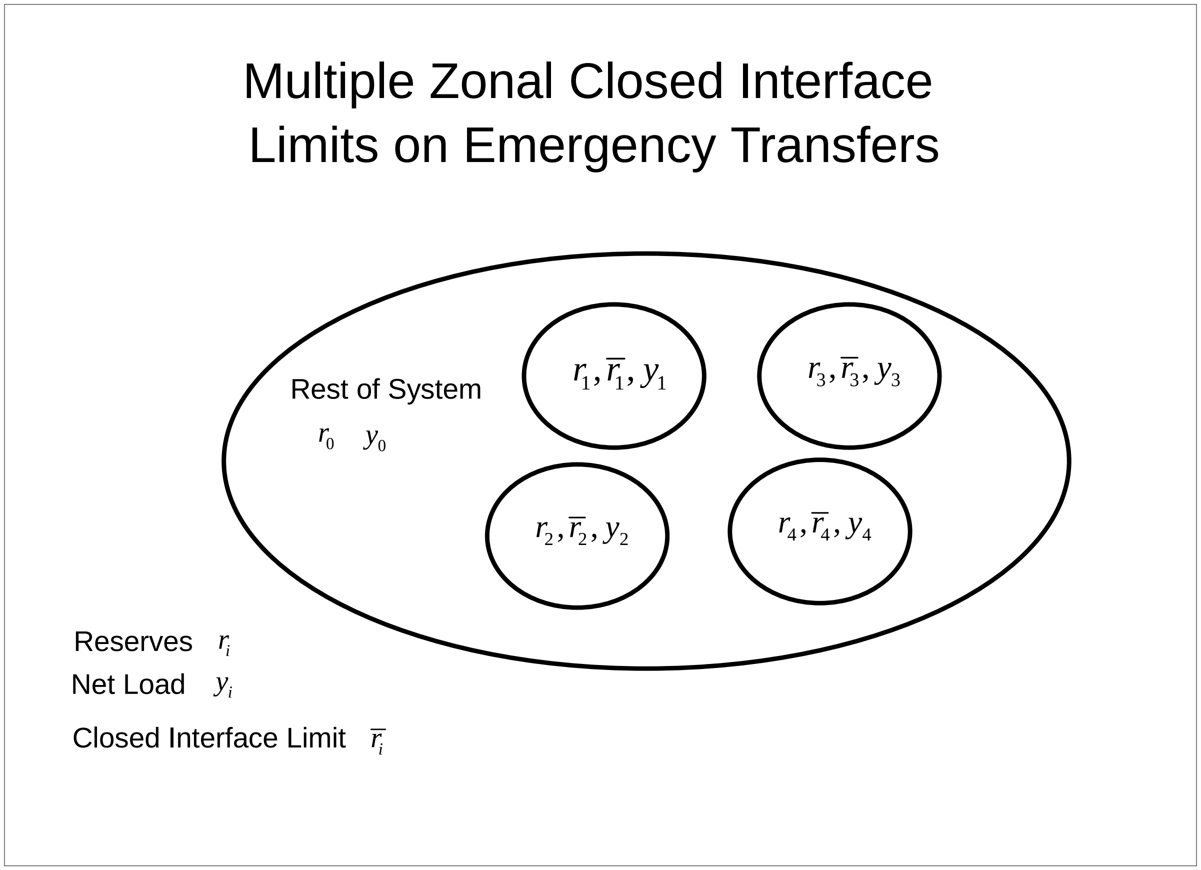
$$p_{\overline{r}_1} = -\frac{\partial ZVEUE\left(r_0,\overline{r}_1,r_1,\overline{r}_2,r_2\right)}{\partial \overline{r}_1} = v_1 P\left(y_1 \geq \overline{r}_1 + r_1\right) - v_0 \begin{bmatrix} P\left(y_0+y_2 \geq r_0+r_2-\overline{r}_1, y_1 \geq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \\ P\left(y_0 \geq r_0-\overline{r}_1-\overline{r}_2, y_1 \geq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \end{bmatrix}$$

$$p_{\overline{r}_2} = -\frac{\partial ZVEUE\left(r_0,\overline{r}_1,r_1,\overline{r}_2,r_2\right)}{\partial \overline{r}_2} = v_2 P\left(y_2 \geq \overline{r}_2 + r_2\right) - v_0 \begin{bmatrix} P\left(y_0+y_1 \geq r_0+r_1-\overline{r}_2, y_1 \leq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \\ P\left(y_0 \geq r_0-\overline{r}_1-\overline{r}_2, y_1 \geq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \end{bmatrix}$$

$$p_{r_0} = -\frac{\partial ZVEUE\left(r_0,\overline{r}_1,r_1,\overline{r}_2,r_2\right)}{\partial r_0} = v_0 \begin{bmatrix} P\left(y_0+y_1+y_2 \geq r_0+r_1+r_2, y_1 \leq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \\ +P\left(y_0+y_2 \geq r_0+r_2-\overline{r}_1, y_1 \geq \overline{r}_1+r_1, y_2 \leq \overline{r}_2+r_2\right) \\ +P\left(y_0+y_1 \geq r_0+r_1-\overline{r}_2, y_1 \leq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \\ +P\left(y_0 \geq r_0-\overline{r}_1-\overline{r}_2, y_1 \geq \overline{r}_1+r_1, y_2 \geq \overline{r}_2+r_2\right) \end{bmatrix}$$

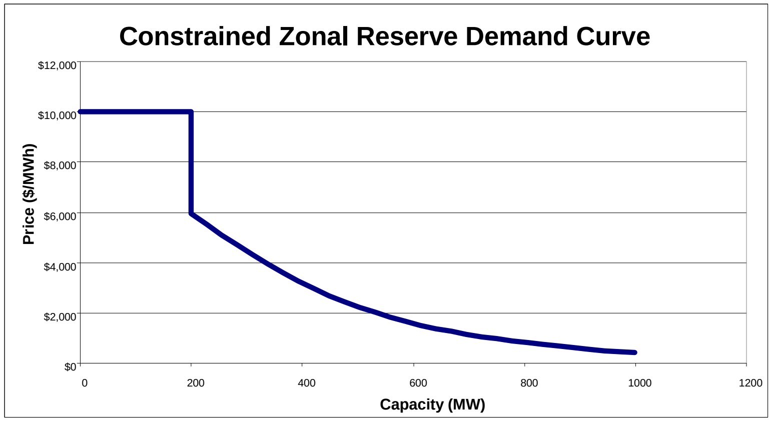
With assumed distributions for the individual net loads, the same benchmarking allows a specification of the implied operating reserves and interface demand curves.

The same principles would apply to specifying the zonal demand curves for any collection of mutually exclusive constrained zones.

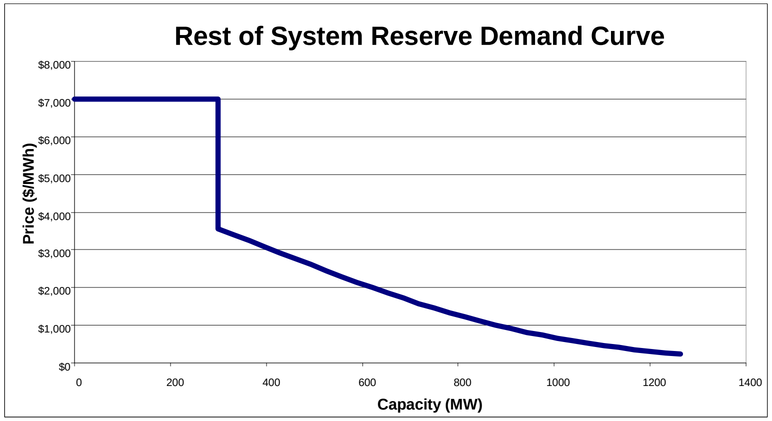


As before, the deterministic approach to security constrained economic dispatch adds lower bounds on the required reserve to ensure that for a set of monitored contingencies (e.g., an n-1 standard) there is sufficient operating reserve to maintain the system for an emergency period.

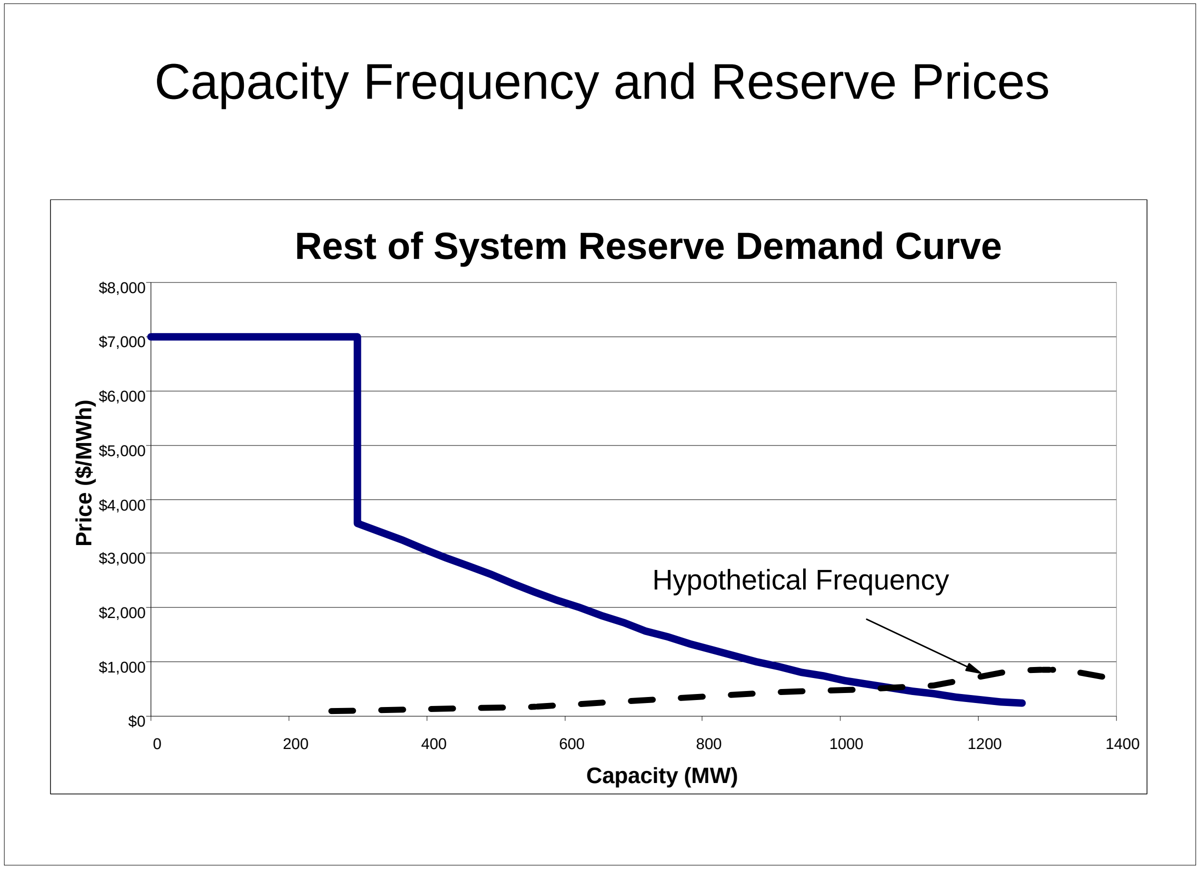
Contingency Constrained Demand for Reserves



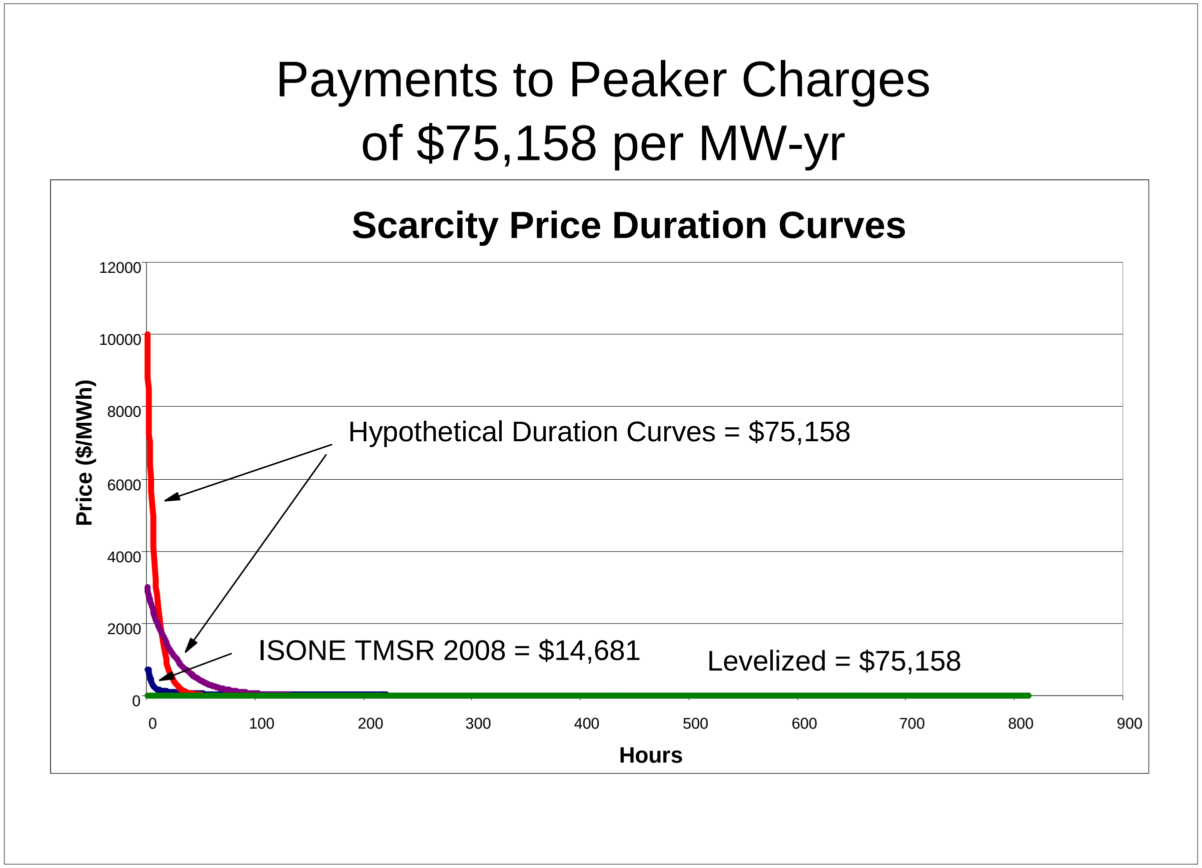
Contingency Constrained Demand for Reserves



An interesting question is the frequency of different reserve levels and the interaction with the operating reserve demand curve. This will determine the scarcity price duration curve.



Different scarcity pricing duration curves will determine the contribution of scarcity prices to total payments for energy and reserves. For example, consider the PJM estimate of a fixed charge for a peaker at $75,158 per MW-yr. The hypotheticals illustrate consistent alternative duration curves. These are compared with the actual 2008 price duration curve in ISONE for ten minute spinning reserves (TMSR) for location ID 7000.



The economic dispatch with zonal model of locational operating reserves is defined implicitly by the operating reserve and interface demand curves.

$$
\begin{aligned}
&\underset{y^0, d^0, g^0, r, \overline{r}, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - ZVEUE\left(r, \overline{r}, u\right) \\
&s.t. \\
&\quad y^0 = d^0 - g^0, \\
&\quad H^0 y^0 \le b^0, \\
&\quad g^0 + r \le u \bullet Cap^0, \\
&\quad \iota^t y^0 = 0, \\
&\quad g^0 \le u \bullet Cap^0 \\
&\quad h\left(y^0\right) + \overline{r} \le Cap^{Interface}.
\end{aligned}
$$

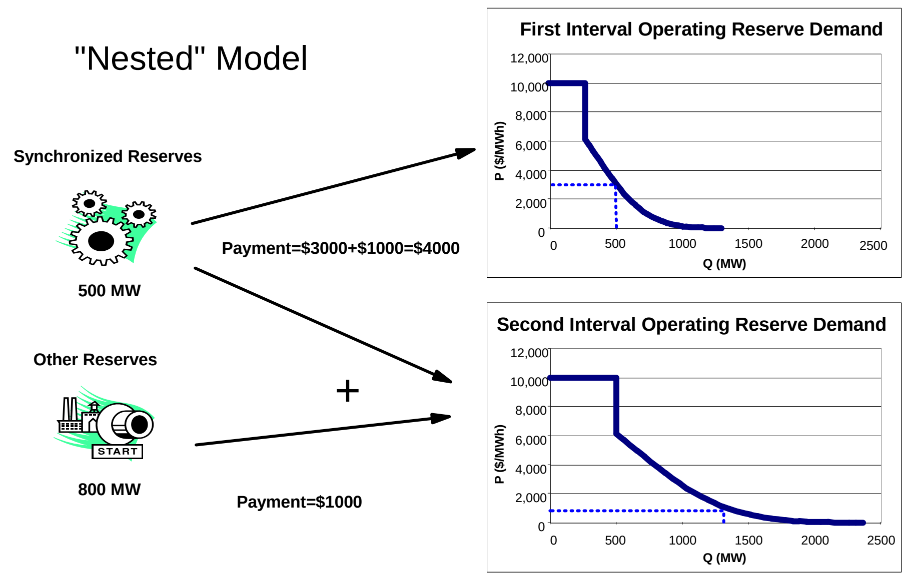
The benchmarked demand curves can be integrated easily and included directly in the economic dispatch model. If necessary, the benchmark values can be adjusted during the solution process to ensure an accurate representation of the final demand prices.[7]

---

7 W. Hogan, "Energy Policy Models for Project Independence," *Computers and Operations Research*, Vol. 2, Pergamon Press, 1975.

Multiple types of operating reserves exist according to response time. A nested model divides the period into consecutive intervals. Reserve schedules set before the period. Uncertainty revealed after the start of the period. Faster responding reserves modeled as available for subsequent intervals. The operating reserve demand curves apply to intervals and the payments to generators include the sum of the prices for the available intervals.

Multiple Operating Reserve Demand Types (Intervals)



Nested model with two intervals, decisions made before uncertainty revealed.

Improved pricing through an explicit operating reserve demand curve raises a number of issues.

Demand Response: Better pricing implemented through the operating reserve demand curve would provide an important signal and incentive for flexible demand participation in spot markets.

Price Spikes: A higher price would be part of the solution. Furthermore, the contribution to the "missing money" from better pricing would involve many more hours and smaller price increases.

Practical Implementation: The NYISO, ISONE and MISO implementations dispose of any argument that it would be impractical to implement an operating reserve demand curve. The only issues are the level of the appropriate price and the preferred model of locational reserves.

Operating Procedures: Implementing an operating reserve demand curve does not require changing the practices of system operators. Reserve and energy prices would be determined simultaneously treating decisions by the operators as being consistent with the adopted operating reserve demand curve.

Multiple Reserves: The demand curve would include different kinds of operating reserves, from spinning reserves to standby reserves.

Reliability: Market operating incentives would be better aligned with reliability requirements.

Market Power: Better pricing would remove ambiguity from analyses of high prices and distinguish (inefficient) economic withholding through high offers from (efficient) scarcity pricing derived from the operating reserve demand curve.

Hedging: The Basic Generation Service auction in New Jersey provides a prominent example that would yield an easy means for hedging small customers with better pricing.

Increased Costs: The higher average energy costs from use of an operating reserve demand curve do not automatically translate into higher costs for customers. In the aggregate, there is an argument that costs would be lower.

Compared to a perfect model, there are many simplifying assumptions needed to specify and operating reserve demand curve. The sketch of the operating reserve demand curve(s) in a network could be extended.

- **Empirical Estimation.** Use existing LOLP models or LOLP extensions with networks to estimate approximate LOLP distributions at nodes.
- **Value of Lost Load.** There are different estimates of lost load. For demand curve estimation the relevant value is the marginal of the average VOLL across the group that would first be curtailed in the event of an outage greater than the available reserves.
- **Multiple Periods.** Incorporate multiple periods of commitment and response time. Handled through the usual supply limits on ramping.
- **Operating Rules.** Incorporate up and down ramp rates, deratings, emergency procedures, etc.
- **Pricing incidence.** Charging participants for impact on operating reserve costs, with any balance included in uplift.[8]
- **Minimum Uplift Pricing.** Dispatch-based pricing that resolves inconsistencies by minimizing the total value of the price discrepancies.
- **…**

---

8 Brendan Kirby and Eric Hirst, "Allocating the Cost of Contingency Reserves," *The Electricity Journal*, December 2003, 99. 39-47.

Supplemental material

- **On design of operating reserve demand curve.**

Begin with an expected value formulation of economic dispatch that might appeal in principle. Given benefit (*B*) and cost (*C*) functions, demand (*d*), generation (*g*), plant capacity (*Cap*), reserves (*r*), commitment decisions (*u*), transmission constraints (*H*), and state probabilities (*p*):

$$\underset{y^i, d^i, g^i, r, u \in \{0,1\}}{Max} \; p_0\left(B^0\left(d^0\right) - C^0\left(g^0, r, u\right)\right) + \sum_{i=1}^{N} p_i\left(B^i\left(d^i, d^0\right) - C^i\left(g^i, g^0, r, u\right)\right)$$

$$s.t.$$

$$y^i = d^i - g^i, \quad i = 0,1,2,\cdots,N,$$

$$\iota^t y^i = 0, \quad i = 0,1,2,\cdots,N,$$

$$H^i y^i \le b^i, \quad i = 0,1,2,\cdots,N,$$

$$g^0 + r \le u \bullet Cap^0,$$

$$g^i \le g^0 + r, \quad i = 1,2,\cdots,N,$$

$$g^i \le u \bullet Cap^i, \quad i = 0,1,2,\cdots,N.$$

Suppose there are *K* possible contingencies. The interesting cases have $K \gg 10^3$. The number of possible system states is $N = 2^K$, or more than the stars in the Milky Way. Some approximation will be in order.[9]

[9] Shams N. Siddiqi and Martin L. Baughman, "Reliability Differentiated Pricing of Spinning Reserve," IEEE Transactions on Power Systems, Vol. 10, No. 3, August 1995, pp.1211-1218. José M. Arroyo and Francisco D. Galiana, "Energy and Reserve Pricing in Security and Network-Constrained Electricity Markets," IEEE Transactions On Power Systems, Vol. 20, No. 2, May 2005, pp. 634-643. François Bouffard, Francisco D. Galiana, and Antonio J. Conejo, "Market-Clearing With Stochastic Security—Part I: Formulation," IEEE Transactions On Power Systems, Vol. 20, No. 4, November 2005, pp. 1818-1826; "Part II: Case Studies," pp. 1827-1835.

Introduce random changes in load ε^i and possible lost load l^i in at least some conditions.

$$\underset{y^i, g^i, l^i, r, u \in \{0,1\}}{Max} \; p_0 \left(B^0 \left(d^0 \right) - C^0 \left(g^0, r, u \right) \right) + \sum_{i=1}^{N} p_i \left(B^i \left(d^o + \varepsilon^i - l^i, d^0 \right) - C^i \left(g^i, g^0, r, u \right) \right)$$

$$\begin{aligned}
s.t.& \\
& y^0 = d^0 - g^0, \\
& y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N, \\
& \iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N, \\
& H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N, \\
& g^0 + r \le u \bullet Cap^0, \\
& g^i \le g^0 + r, \quad i = 1, 2, \cdots, N, \\
& g^i \le u \bullet Cap^i, \quad i = 0, 1, 2, \cdots, N.
\end{aligned}$$

Simplify the benefit and cost functions:

$$B^i \left(d^o + \varepsilon^i - l^i, d^0 \right) \approx B^0 \left(d^0 \right) + k_d^i - v^t l^i \quad , \qquad C^i \left(g^i, g^0, r, u \right) \approx C^0 \left(g^0, r, u \right) + k_g^i .$$

This produces an approximate objective function:

$$p_0 \left(B^0 \left(d^0 \right) - C^0 \left(g^0, r, u \right) \right) + \sum_{i=1}^{N} p_i \left(B^i \left(d^o - l^i, d^0 \right) - C^i \left(g^i, g^0, r, u \right) \right) = B^0 \left(d^0 \right) - C^0 \left(g^0, r, u \right) + \sum_{i=1}^{N} p_i \left(k_d^i - k_g^i \right) - v^t \sum_{i=1}^{N} p_i l^i .$$

The revised formulation highlights the pre-contingency objective function and the role of the value of the expected undeserved energy.

$$\underset{y^i, g^i, l^i, r, u \in \{0,1\}}{Max} B^0\left(d^0\right) - C^0\left(g^0, r, u\right) - v^t \sum_{i=1}^{N} p_i l^i$$

$$\begin{aligned}
&s.t.\\
&\quad y^0 = d^0 - g^0,\\
&\quad y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N,\\
&\quad \iota^t y^i = 0, \quad i = 0, 1, 2, \cdots, N,\\
&\quad H^i y^i \le b^i, \quad i = 0, 1, 2, \cdots, N,\\
&\quad g^0 + r \le u \bullet Cap^0,\\
&\quad g^i \le g^0 + r, \quad i = 1, 2, \cdots, N,\\
&\quad g^i \le u \bullet Cap^i, \quad i = 0, 1, 2, \cdots, N.
\end{aligned}$$

There are still too many system states.

Define the optimal value of expected unserved energy (VEUE) as the result of all the possible optimal post-contingency responses given the pre-contingency commitment and scheduling decisions.

$$VEUE\left(d^0, g^0, r, u\right) = \underset{y^i, g^i, l^i}{Min}\, v^t \sum_{i=1}^{N} p_i l^i$$

$$s.t.$$

$$y^i = d^0 + \varepsilon^i - g^i - l^i, \quad i = 1, 2, \cdots, N,$$

$$\iota^t y^i = 0, \quad i = 1, 2, \cdots, N,$$

$$H^i y^i \leq b^i, \quad i = 1, 2, \cdots, N,$$

$$g^i \leq g^0 + r, \quad i = 1, 2, \cdots, N,$$

$$g^i \leq u \bullet Cap^i, \quad i = 1, 2, \cdots, N.$$

This second stage problem subsumes all the redispatch and curtailment decisions over the operating period after the commitment and scheduling decisions.

The expected value formulation reduces to a much more manageable scale with the introduction of the implicit VEUE function.

$$\begin{aligned}
&\underset{y^0,d^0,g^0,r,u\in\{0,1\}}{Max}\ B^0\left(d^0\right)-C^0\left(g^0,r,u\right)-VEUE\left(d^0,g^0,r,u\right)\\
&s.t.\\
&\quad y^0=d^0-g^0,\\
&\quad H^0y^0\le b^0,\\
&\quad g^0+r\le u\bullet Cap^0,\\
&\quad \iota^t y^0=0,\\
&\quad g^0\le u\bullet Cap^0.
\end{aligned}$$

The optimal value of expected unserved energy defines the demand for operating reserves. This formulation of the problem follows the outline of existing operating models except for the exclusion of contingency constraints.

The probability calculation for the constrained zone in the zonal model includes the following key element:[10]

$$P\left(y_a + y_b \ge k_1 \middle| y_b = x_b\right) = P\left(y_a + x_b \ge k_1 \middle| y_b = x_b\right) = P\left(y_a + x_b \ge k_1\right) = 1 - F_a\left(k_1 - x_b\right)$$
$$P\left(y_a \le x_b \middle| y_b \le k_2\right) = F_b\left(x_b\right) / F_b\left(k_2\right)$$
$$f_{y_b | y_b \le k_2}\left(y_b\right) = f_b\left(y_b\right) / F_b\left(k_2\right)$$
$$P\left(y_a + y_b \ge k_1 \middle| y_b \le k_2\right) = \int_{-\infty}^{k_2} P\left(y_a + x_b \ge k_1 \middle| y_b = x_b\right) f_{y_b | y_b \le k_2}\left(x_b\right) dx_b$$
$$= \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) / F_b\left(k_2\right) dx_b = \frac{1}{F_b\left(k_2\right)} \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$
$$P\left(y_a + y_b \ge k_1, y_b \le k_2\right) = P\left(y_a + y_b \ge k_1 \middle| y_b \le k_2\right) P\left(y_b \le k_2\right)$$
$$P\left(y_a + y_b \ge k_1, y_b \le k_2\right) = \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$

Hence

$$P\left(y_a + y_b \ge k_1, y_b \le k_2\right) = \int_{-\infty}^{k_2} \left[1 - F_a\left(k_1 - x_b\right)\right] f_b\left(x_b\right) dx_b$$
$$P\left(y_0 + y_1 \ge r_0 + r_1, y_1 \le \overline{r}_1 + r_1\right) = \int_{-\infty}^{\overline{r}_1 + r_1} \left[1 - F_0\left(r_0 + r_1 - x_1\right)\right] f_1\left(x_1\right) dx_1$$

10 Thanks to Alberto Abadie for the probability tutorial.

The probability calculation for the rest of system in the zonal model includes the following key element:

$$P\left(y_a + y_b \geq k_1 \middle| y_a = x_a\right) = P\left(x_a + y_b \geq k_1 \middle| y_a = x_a\right) = P\left(x_a + y_b \geq k_1\right) = 1 - F_b\left(k_1 - x_a\right)$$

$$P\left(y_a \leq x_a \middle| y_a \geq k_2\right) = F_a\left(x_a\right) \big/ \left[1 - F_a\left(k_2\right)\right]$$

$$f_{y_a|y_a \geq k_2}\left(y_a\right) = f_a\left(y_a\right) \big/ \left[1 - F_a\left(k_2\right)\right]$$

$$P\left(y_a + y_b \geq k_1 \middle| y_a \geq k_2\right) = \int_{k_2}^{\infty} P\left(y_a + y_b \geq k_1 \middle| y_a = x_a\right) f_{y_a|y_a \geq k_2}\left(x_a\right) dx_a$$

$$= \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) \big/ \left[1 - F_a\left(k_2\right)\right] dx_a = \frac{1}{1 - F_a\left(k_2\right)} \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = P\left(y_a + y_b \geq k_1 \middle| y_a \geq k_2\right) P\left(y_a \geq k_2\right)$$

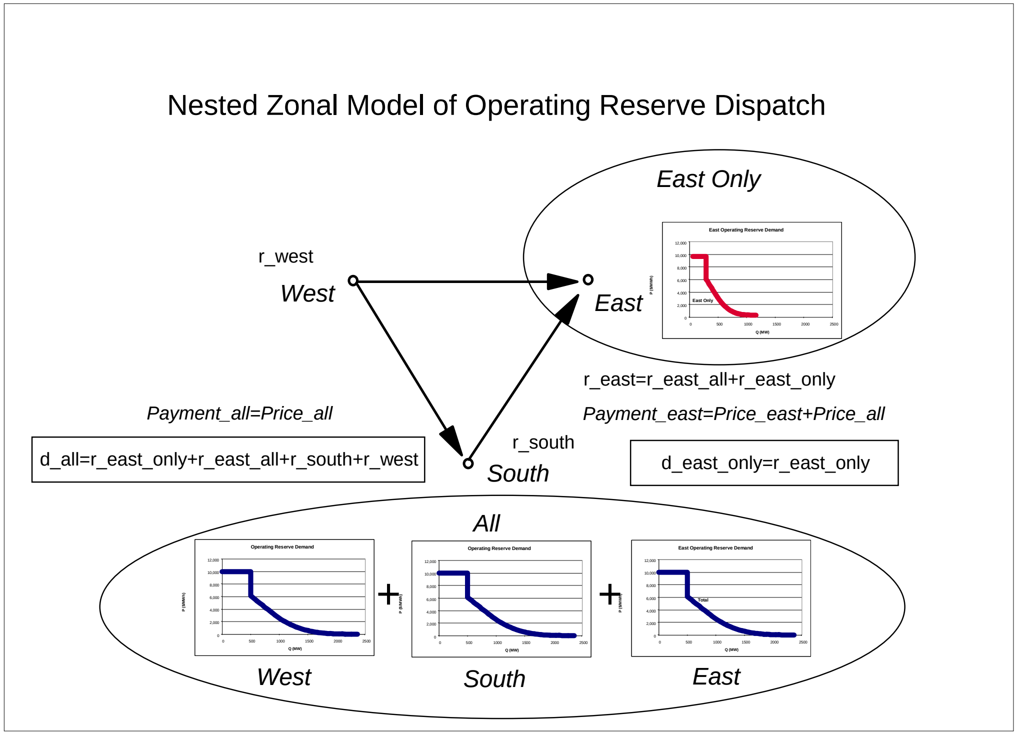
$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

Hence.

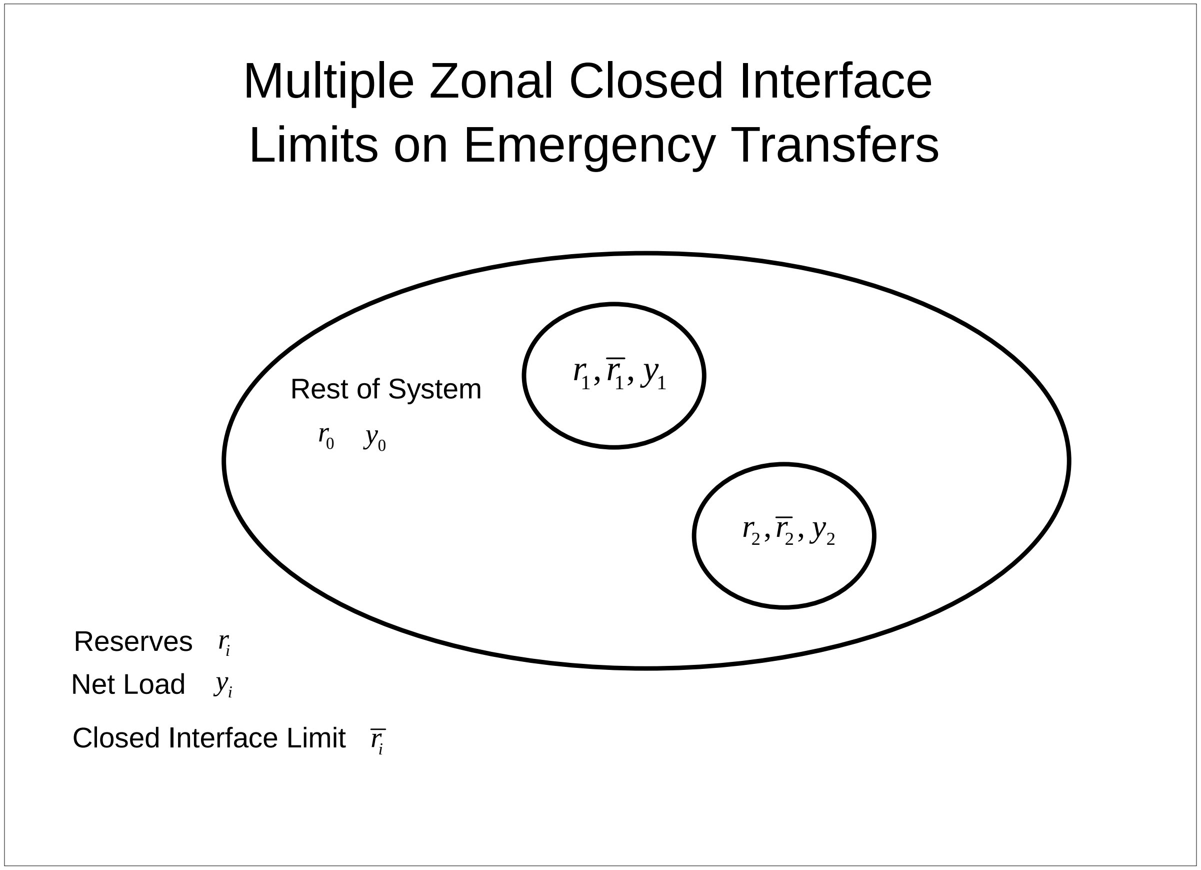
$$P\left(y_a + y_b \geq k_1, y_a \geq k_2\right) = \int_{k_2}^{\infty} \left[1 - F_b\left(k_1 - x_a\right)\right] f_a\left(x_a\right) dx_a$$

$$P\left(y_0 + y_1 \geq r_0 + r_1, y_0 \geq r_0 - \overline{r}_1\right) = \int_{r_0 - \overline{r}_1}^{\infty} \left[1 - F_1\left(r_0 + r_1 - x_0\right)\right] f_0\left(x_0\right) dx_0$$

The nested model of simultaneous dispatch of locational operating reserves and energy is used in NYISO, ISONE, and MISO. This model must derive from a different characterization of the zonal constraints. A zonal model analogous to the long-term reserve requirements approach produces interactions among regions but not in the same was as assumed in this cascade or nested formulation.

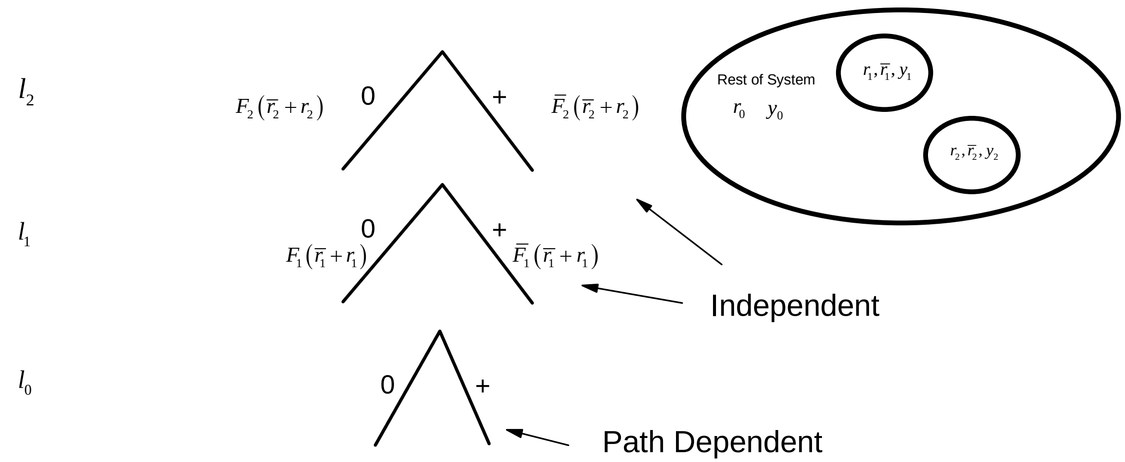


The case of multiple constrained zones is a natural extension of the case for a single constrained zone.



The probability of losses depends on the path of binding interface constraints.

Loss of Load Probability Structure

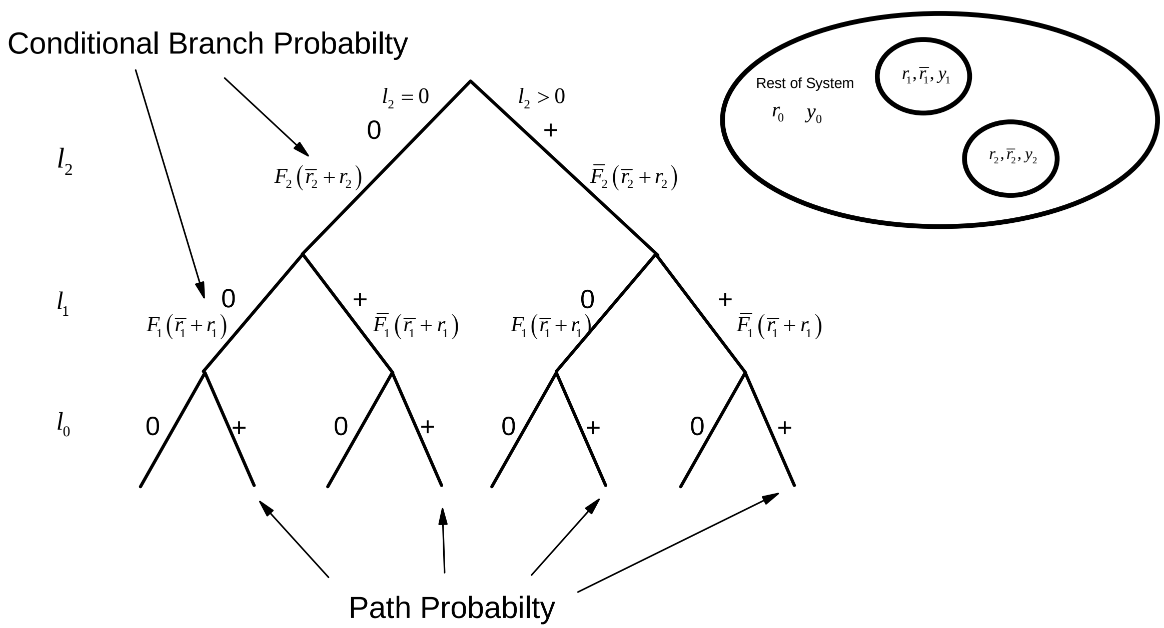


$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\, dx_i$$

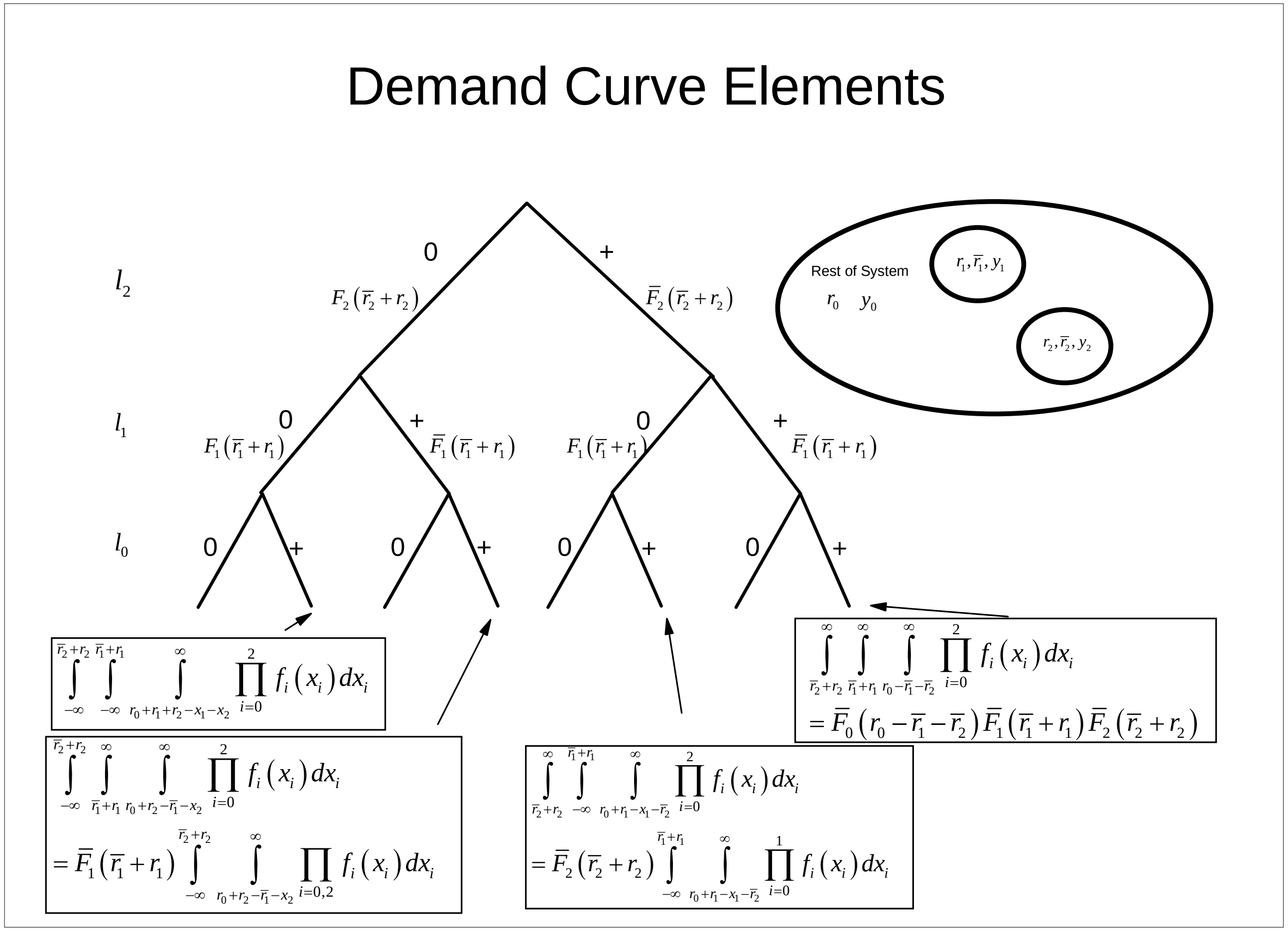
The probability tree captures the dependencies of loss of load.

Loss of Load Probabilities

$$ZVEUE\left(r_0,\bar{r}_1,r_1,\bar{r}_2,r_2\right)=E_y\left[\underset{l_i\geq 0}{Min}\ \left\{v_0 l_0+v_1 l_1+v_2 l_2 \middle| y_0+y_1+y_2-l_0-l_1-l_2\leq r_0+r_1+r_2,\ y_1-l_1\leq \bar{r}_1+r_1,\ y_2-l_2\leq \bar{r}_2+r_2\right\}\right]$$



The loss of load probability structure defines the demand curve elements.



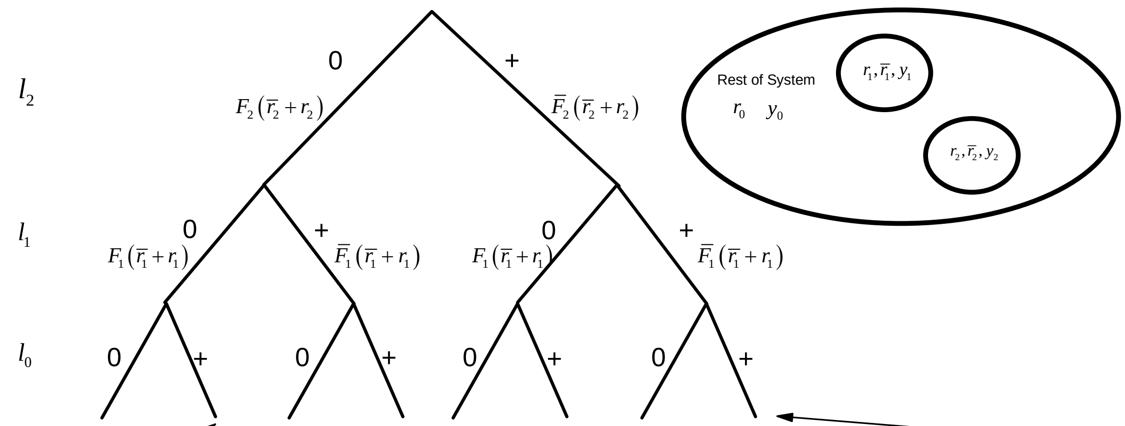
The tree structure identifies the loss probability dependencies and the paths where incremental capacity affects the losses.

- **Outages and Demand Changes.** The zonal convolutions of capacity outages and demand changes determine the (assumed independent) elementary zonal probability distributions of changes in net load.
- **Tree Structure.** The dependencies for losses and binding interface constraints defined by the probability tree structure determine the path probabilities for loss of load in each location as a function of the underlying independent elementary distributions.
- **Demand Curve.** The demand curve is determined by the value of lost load in each zone and the dependencies in the tree structure determining when reserves or interface capacity would be substitutable for losses.
  - **Value of Loss Load.** Assume embedded zones have higher incremental values of lost load.
  - **Substitution of Capacity.** Identify substitution possibilities on alternative paths for zonal losses and binding constraints. For example:
    - **Zonal Losses.** Apply only when interface constraint is binding.
    - **Reserve Substitution.** Higher level reserves substitute for lower level losses only when interface constraint is not binding.
    - **Interface Capacity.** Increased interface capacity for binding interface substitutes lower level losses for higher level losses.

The loss outcomes determine demand for rest of system operating reserve.

Demand Curve Elements: Rest of System

$$p_{r_0} = v_o \left[\int_{-\infty}^{\bar r_2 + r_2} \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar F_1(\bar r_1 + r_1) \int_{-\infty}^{\bar r_2 + r_2} \int_{r_0+r_2-\bar r_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i + \bar F_2(\bar r_2 + r_2) \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0+r_1-x_1-\bar r_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i + \bar F_0(r_0 - \bar r_1 - \bar r_2)\bar F_1(\bar r_1 + r_1)\bar F_2(\bar r_2 + r_2) \right]$$



✓ $$\int_{-\infty}^{\bar r_2 + r_2} \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i$$

✓ $$\int_{-\infty}^{\bar r_2 + r_2} \int_{\bar r_1 + r_1}^{\infty} \int_{r_0+r_2-\bar r_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_1(\bar r_1 + r_1) \int_{-\infty}^{\bar r_2 + r_2} \int_{r_0+r_2-\bar r_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i$$

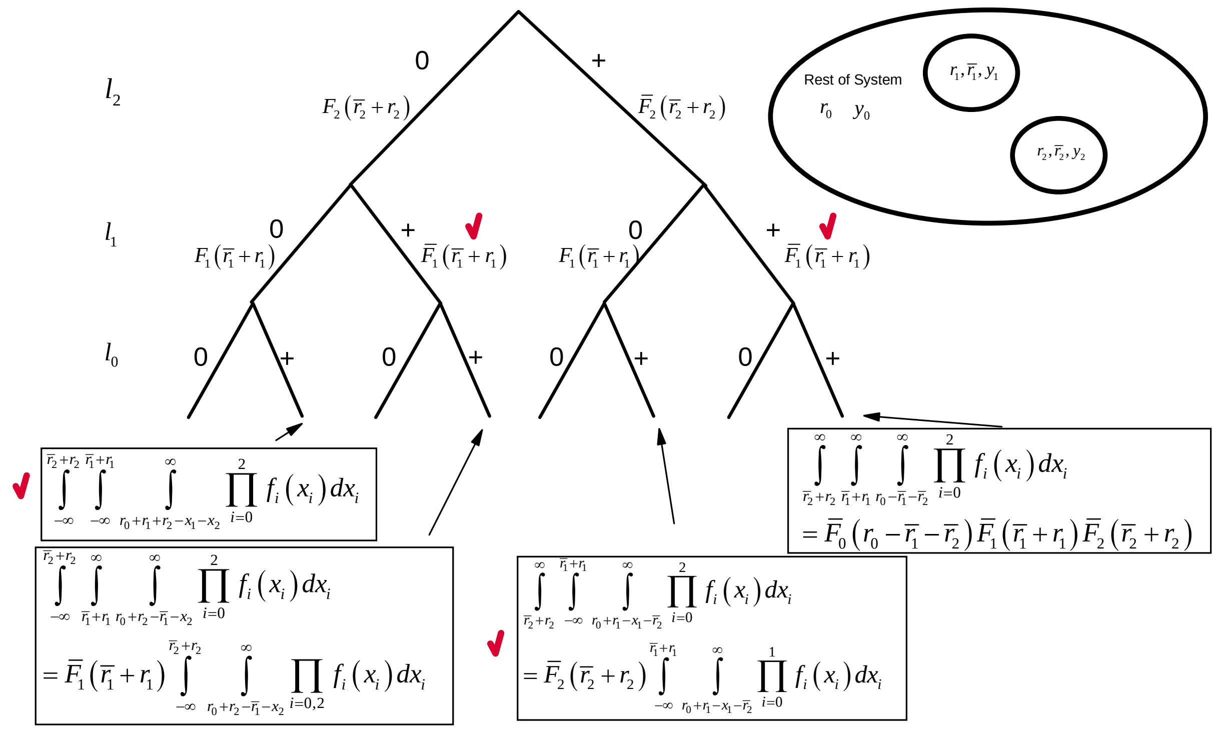
✓ $$\int_{\bar r_2 + r_2}^{\infty} \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0+r_1-x_1-\bar r_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_2(\bar r_2 + r_2) \int_{-\infty}^{\bar r_1 + r_1} \int_{r_0+r_1-x_1-\bar r_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i$$

✓ $$\int_{\bar r_2 + r_2}^{\infty} \int_{\bar r_1 + r_1}^{\infty} \int_{r_0-\bar r_1-\bar r_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_0(r_0 - \bar r_1 - \bar r_2)\bar F_1(\bar r_1 + r_1)\bar F_2(\bar r_2 + r_2)$$

The loss outcomes and dependencies determine the demand for zone 1 operating reserves.

Demand Curve Elements: Zone 1

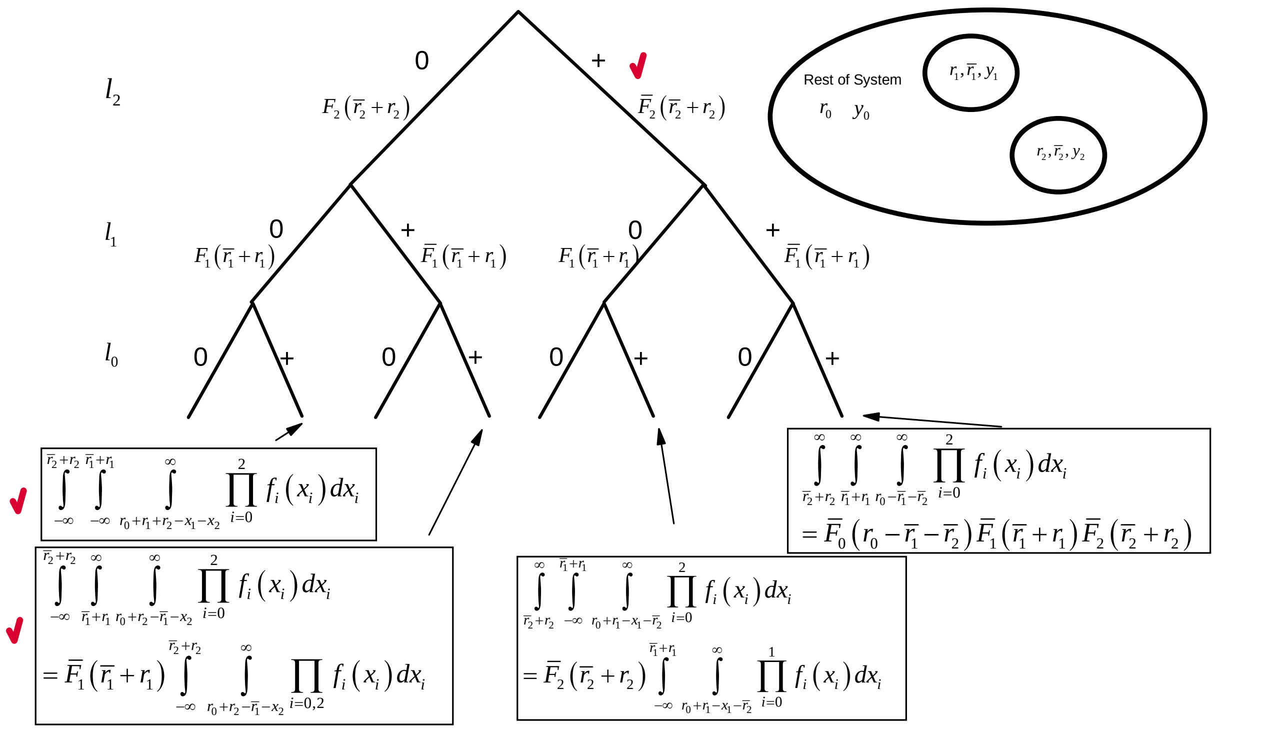
$$p_{r_r} = v_1 \bar{F}_1(\bar{r}_1 + r_1) + v_0 \left[\int_{-\infty}^{\bar{r}_2+r_2} \int_{-\infty}^{\bar{r}_1+r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar{F}_2(\bar{r}_2+r_2) \int_{-\infty}^{\bar{r}_1+r_1} \int_{r_0+r_1-x_1-\bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i \right]$$



The loss outcomes and dependencies determine the demand for zone 2 operating reserves.

Demand Curve Elements: Zone 2

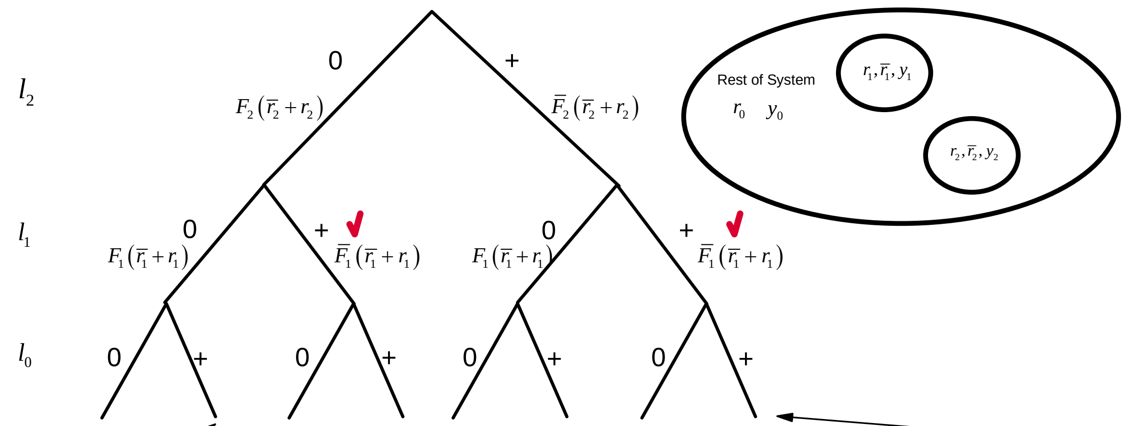
$$p_{r_2} = v_2 \overline{F}_2(\overline{r}_2 + r_2) + v_0 \left[\int_{-\infty}^{\overline{r}_2+r_2} \int_{-\infty}^{\overline{r}_1+r_1} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \overline{F}_1(\overline{r}_1 + r_1) \int_{-\infty}^{\overline{r}_2+r_2} \int_{r_0+r_2-\overline{r}_1-x_2}^{\infty} \prod_{i=0,2} f_i(x_i)\,dx_i \right]$$



The loss outcomes and dependencies determine the demand for zone 1 interface capacity.

Demand Curve Elements: Interface 1

$$p_{\bar{r}_1} = v_1 \overline{F}_1(\bar{r}_1 + r_1) - v_0\left[\overline{F}_1(\bar{r}_1 + r_1)\int_{-\infty}^{\bar{r}_2 + r_2}\int_{r_0 + r_2 - \bar{r}_1 - x_2}^{\infty}\prod_{i=0,2} f_i(x_i)\,dx_i + \overline{F}_0(r_0 - \bar{r}_1 - \bar{r}_2)\overline{F}_1(\bar{r}_1 + r_1)\overline{F}_2(\bar{r}_2 + r_2)\right]$$



$$\int_{-\infty}^{\bar{r}_2 + r_2}\int_{-\infty}^{\bar{r}_1 + r_1}\int_{r_0 + r_1 + r_2 - x_1 - x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i$$

$$\int_{\bar{r}_2 + r_2}^{\infty}\int_{\bar{r}_1 + r_1}^{\infty}\int_{r_0 - \bar{r}_1 - \bar{r}_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_0(r_0 - \bar{r}_1 - \bar{r}_2)\overline{F}_1(\bar{r}_1 + r_1)\overline{F}_2(\bar{r}_2 + r_2)$$

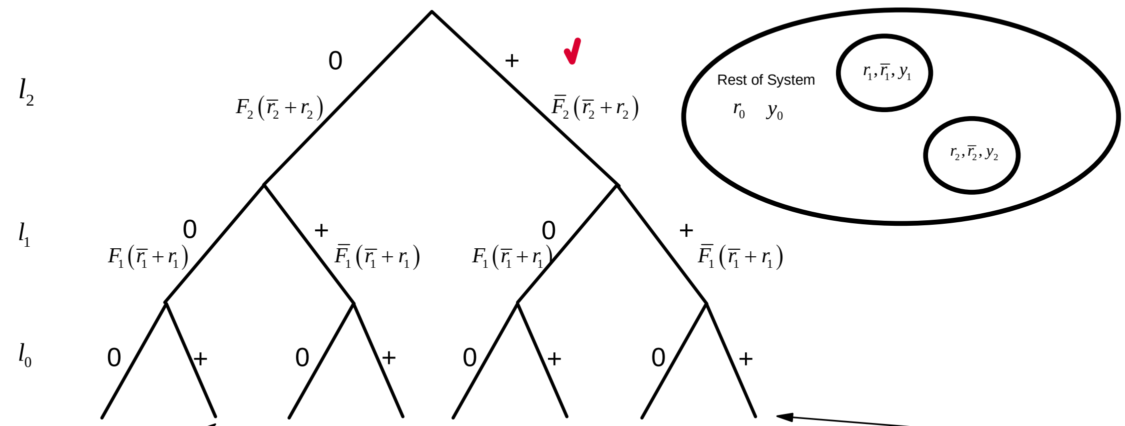
$$\int_{-\infty}^{\bar{r}_2 + r_2}\int_{\bar{r}_1 + r_1}^{\infty}\int_{r_0 + r_2 - \bar{r}_1 - x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_1(\bar{r}_1 + r_1)\int_{-\infty}^{\bar{r}_2 + r_2}\int_{r_0 + r_2 - \bar{r}_1 - x_2}^{\infty}\prod_{i=0,2} f_i(x_i)\,dx_i$$

$$\int_{\bar{r}_2 + r_2}^{\infty}\int_{-\infty}^{\bar{r}_1 + r_1}\int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \overline{F}_2(\bar{r}_2 + r_2)\int_{-\infty}^{\bar{r}_1 + r_1}\int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$

The loss outcomes and dependencies determine the demand for zone 2 interface capacity.

Demand Curve Elements: Interface 2

$$p_{\bar{r}_2} = v_2 \bar{F}_2\left(\bar{r}_2 + r_2\right) - v_0 \left[\bar{F}_2\left(\bar{r}_2 + r_2\right) \int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i + \bar{F}_0\left(r_0 - \bar{r}_1 - \bar{r}_2\right) \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_2\left(\bar{r}_2 + r_2\right) \right]$$



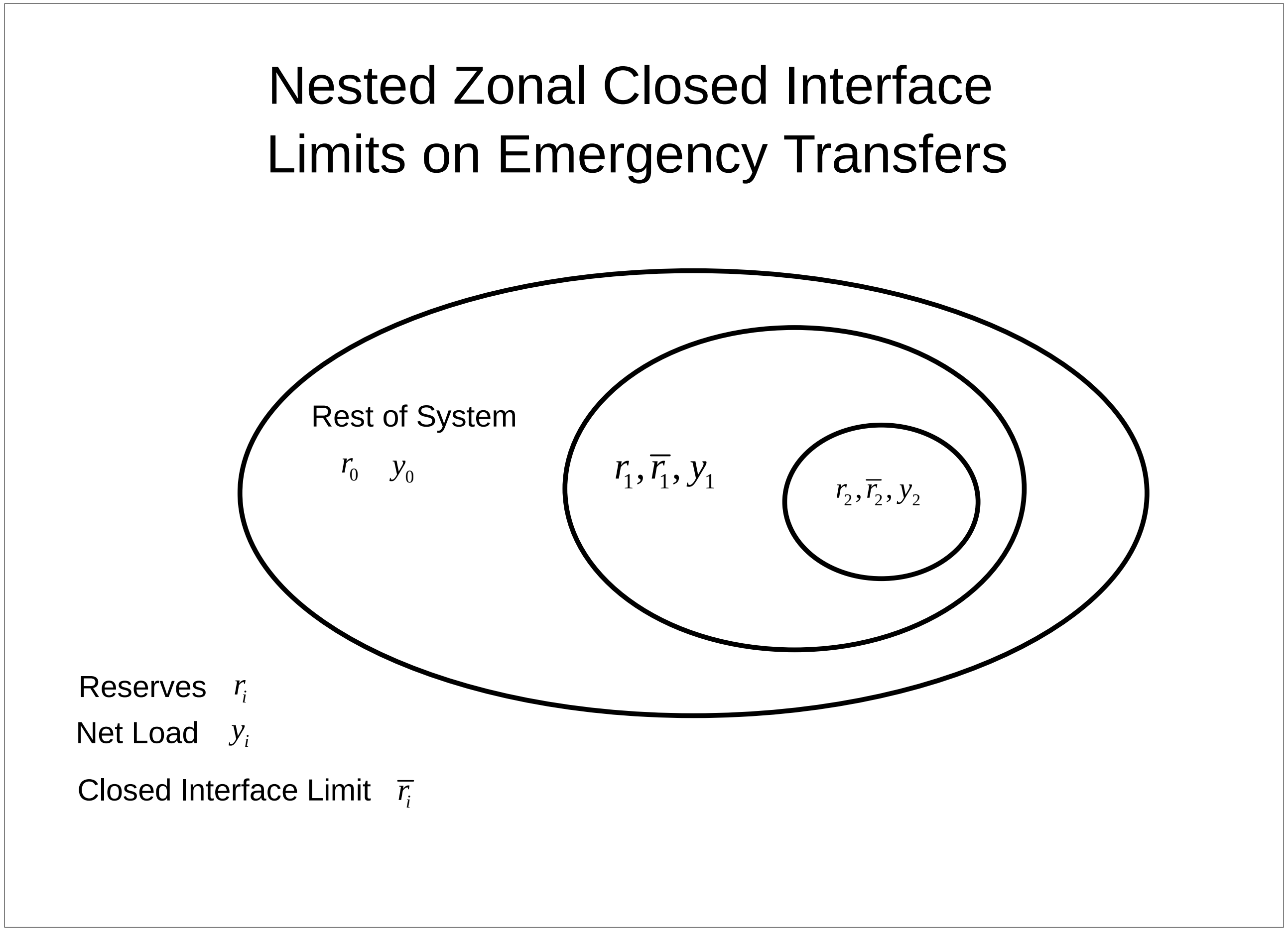
$$\int_{-\infty}^{\bar{r}_2 + r_2} \int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 + r_2 - x_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$

$$\int_{-\infty}^{\bar{r}_2 + r_2} \int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 + r_2 - \bar{r}_1 - x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_1\left(\bar{r}_1 + r_1\right) \int_{-\infty}^{\bar{r}_2 + r_2} \int_{r_0 + r_2 - \bar{r}_1 - x_2}^{\infty} \prod_{i=0,2} f_i\left(x_i\right) dx_i$$

$$\int_{\bar{r}_2 + r_2}^{\infty} \int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_2\left(\bar{r}_2 + r_2\right) \int_{-\infty}^{\bar{r}_1 + r_1} \int_{r_0 + r_1 - x_1 - \bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i$$

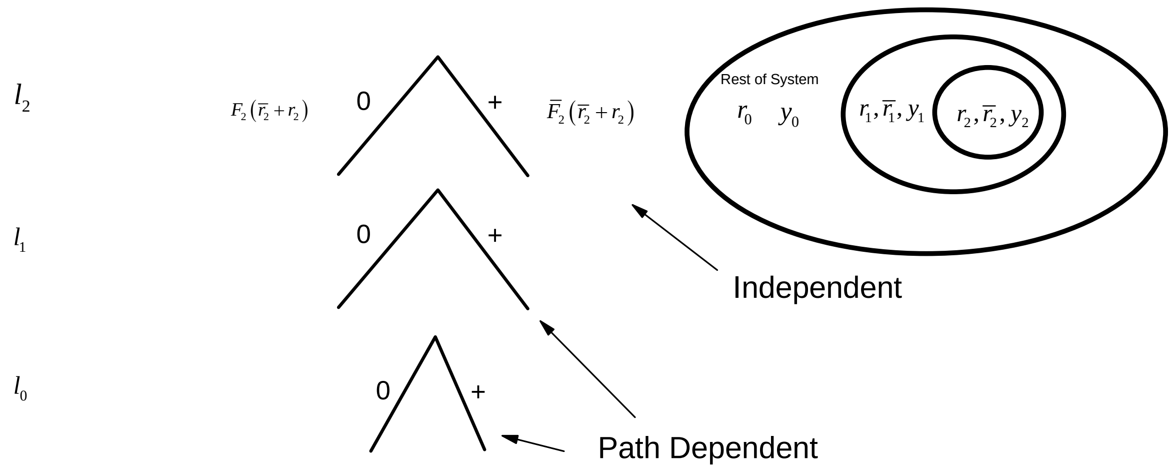
$$\int_{\bar{r}_2 + r_2}^{\infty} \int_{\bar{r}_1 + r_1}^{\infty} \int_{r_0 - \bar{r}_1 - \bar{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_0\left(r_0 - \bar{r}_1 - \bar{r}_2\right) \bar{F}_1\left(\bar{r}_1 + r_1\right) \bar{F}_2\left(\bar{r}_2 + r_2\right)$$

Nested constrained zones define an alternative extension of the case for a single constrained zone.



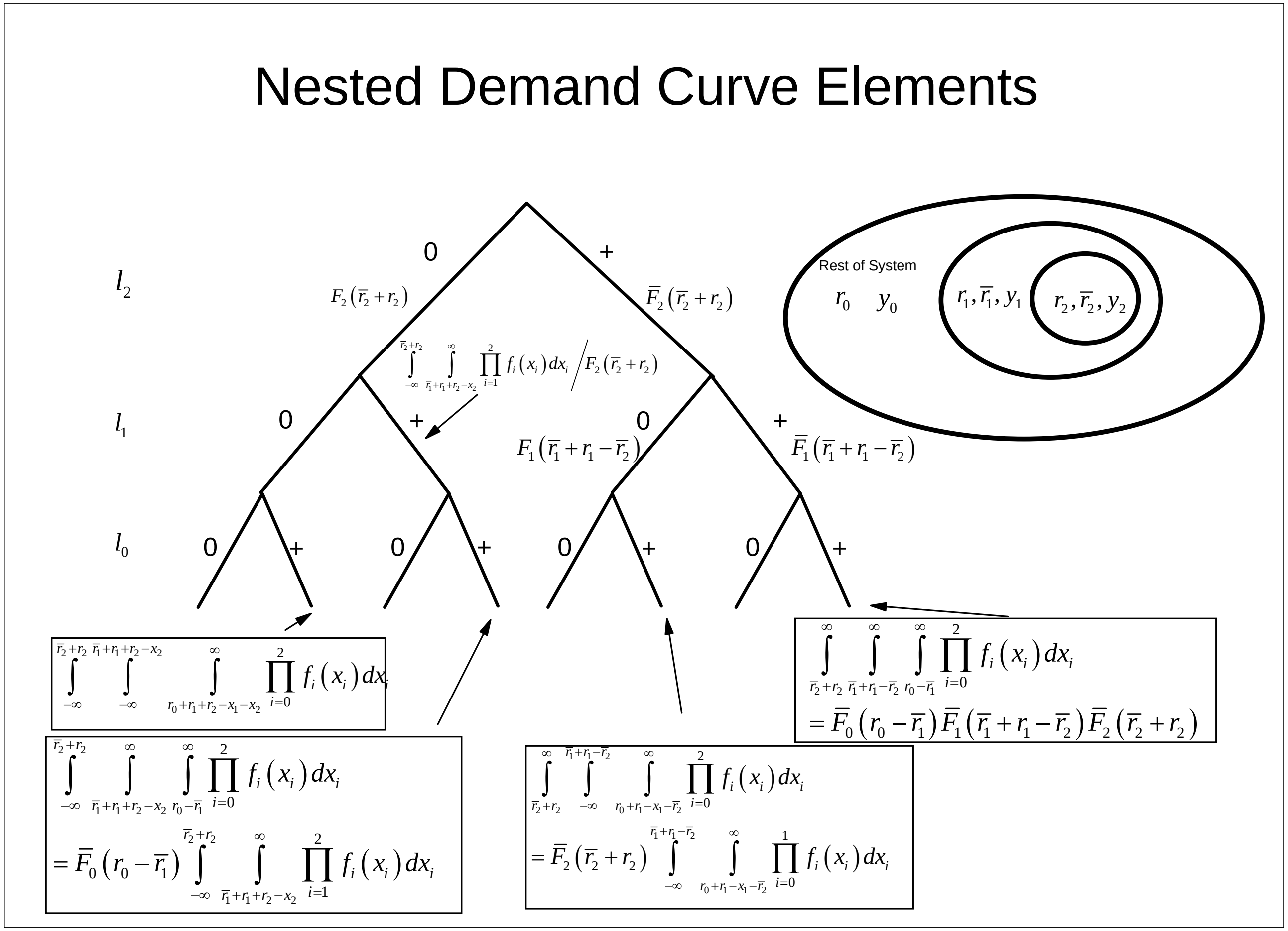
The probability tree for the nested zones captures the dependencies of loss of load.

Nested Loss of Load Probability Structure



$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

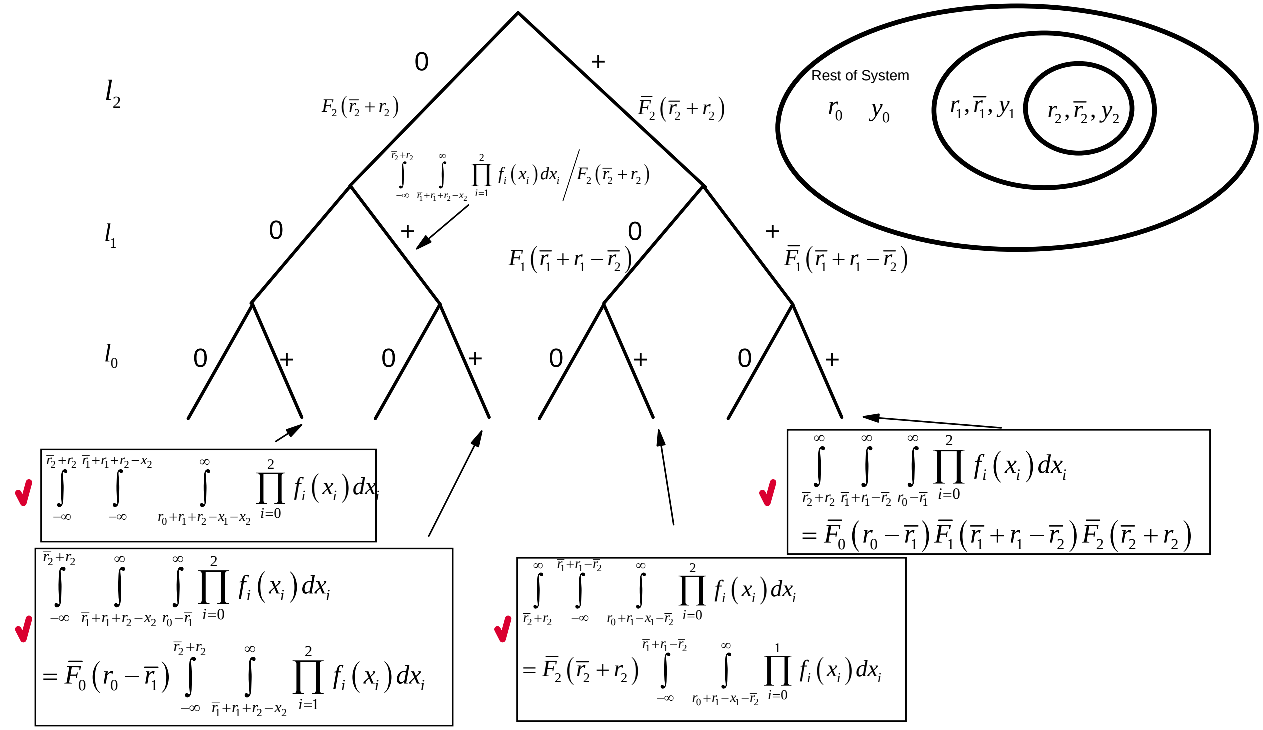
The nested loss of load probability structure defines the demand curve elements.



The nested loss outcomes and dependencies determine the demand for rest of system operating reserves.

Nested Demand Curve Elements: Rest of System

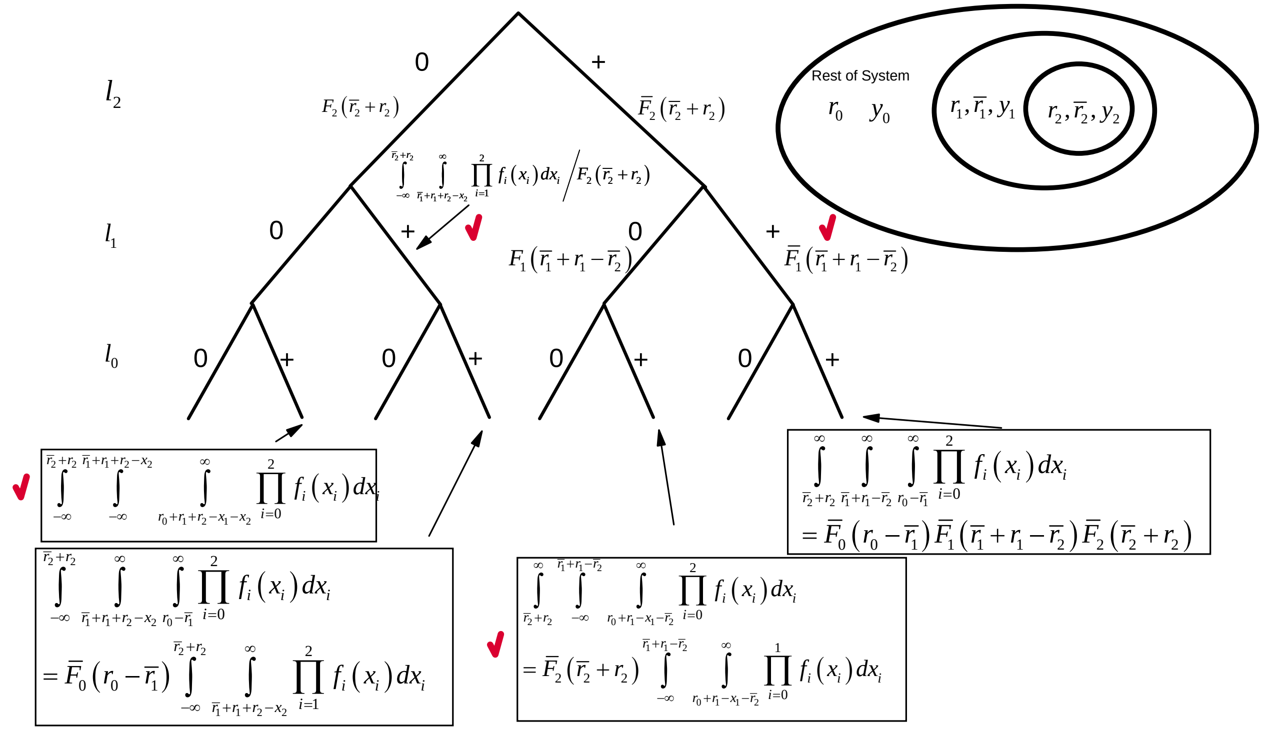
$$p_{r_0} = v_0\left[\int_{-\infty}^{\bar r_2+r_2}\int_{-\infty}^{\bar r_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)dx_i + \bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)dx_i + \bar F_2(\bar r_2+r_2)\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)dx_i + \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right]$$



The nested loss outcomes and dependencies determine the demand for zone 1 operating reserves.

Nested Demand Curve Elements: Zone 1

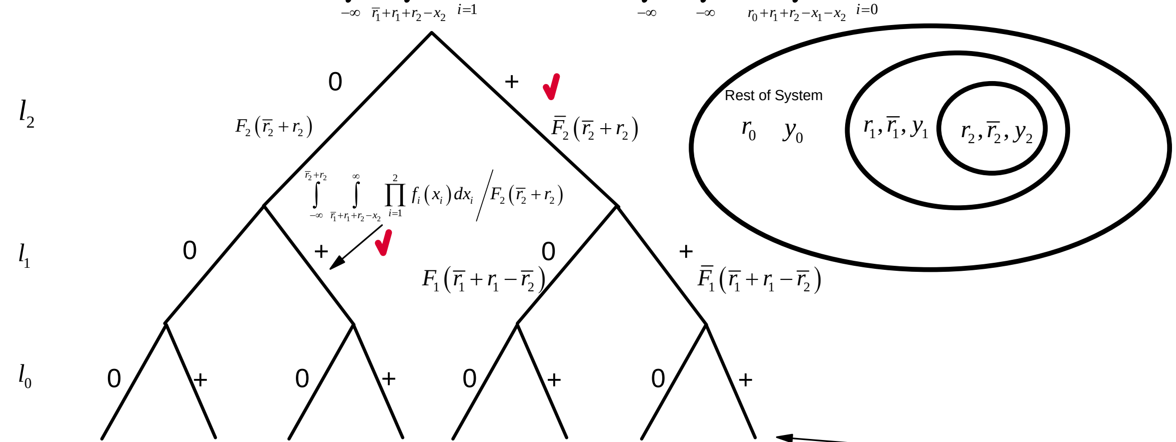
$$p_{r_1} = v_1 \left[\int_{-\infty}^{\bar{r}_2+r_2} \int_{\bar{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar{F}_1(\bar{r}_1+r_1-\bar{r}_2)\bar{F}_2(\bar{r}_2+r_2) \right] + v_0 \left[\int_{-\infty}^{\bar{r}_2+r_2} \int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i(x_i)\,dx_i + \bar{F}_2(\bar{r}_2+r_2) \int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2} \int_{r_0+r_1-x_1-\bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i(x_i)\,dx_i \right]$$



The nested loss outcomes and dependencies determine the demand for zone 2 operating reserves.

Nested Demand Curve Elements: Zone 2

$$p_{r_2} = v_2 \bar{F}_2\left(\bar{r}_2 + r_2\right) + v_1 \int_{-\infty}^{\bar{r}_2+r_2} \int_{\bar{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i\left(x_i\right) dx_i + v_0 \int_{-\infty}^{\bar{r}_2+r_2} \int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$



$$\int_{-\infty}^{\bar{r}_2+r_2} \int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2} \int_{r_0+r_1+r_2-x_1-x_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i$$

$$\int_{-\infty}^{\bar{r}_2+r_2} \int_{\bar{r}_1+r_1+r_2-x_2}^{\infty} \int_{r_0-\bar{r}_1}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_0\left(r_0 - \bar{r}_1\right) \int_{-\infty}^{\bar{r}_2+r_2} \int_{\bar{r}_1+r_1+r_2-x_2}^{\infty} \prod_{i=1}^{2} f_i\left(x_i\right) dx_i$$

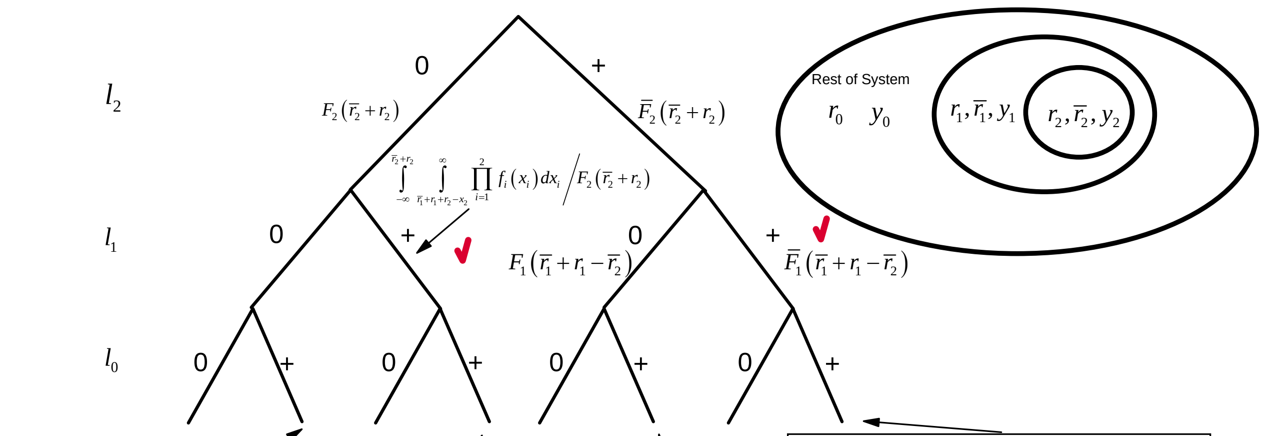
$$\int_{\bar{r}_2+r_2}^{\infty} \int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2} \int_{r_0+r_1-x_1-\bar{r}_2}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_2\left(\bar{r}_2 + r_2\right) \int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2} \int_{r_0+r_1-x_1-\bar{r}_2}^{\infty} \prod_{i=0}^{1} f_i\left(x_i\right) dx_i$$

$$\int_{\bar{r}_2+r_2}^{\infty} \int_{\bar{r}_1+r_1-\bar{r}_2}^{\infty} \int_{r_0-\bar{r}_1}^{\infty} \prod_{i=0}^{2} f_i\left(x_i\right) dx_i = \bar{F}_0\left(r_0 - \bar{r}_1\right) \bar{F}_1\left(\bar{r}_1 + r_1 - \bar{r}_2\right) \bar{F}_2\left(\bar{r}_2 + r_2\right)$$

The nested loss outcomes and dependencies determine the demand for interface 1 capacity.

Nested Demand Curve Elements: Interface 1

$$p_{\bar r_1} = v_1\left[\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right] - v_0\left[\bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i + \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)\right]$$



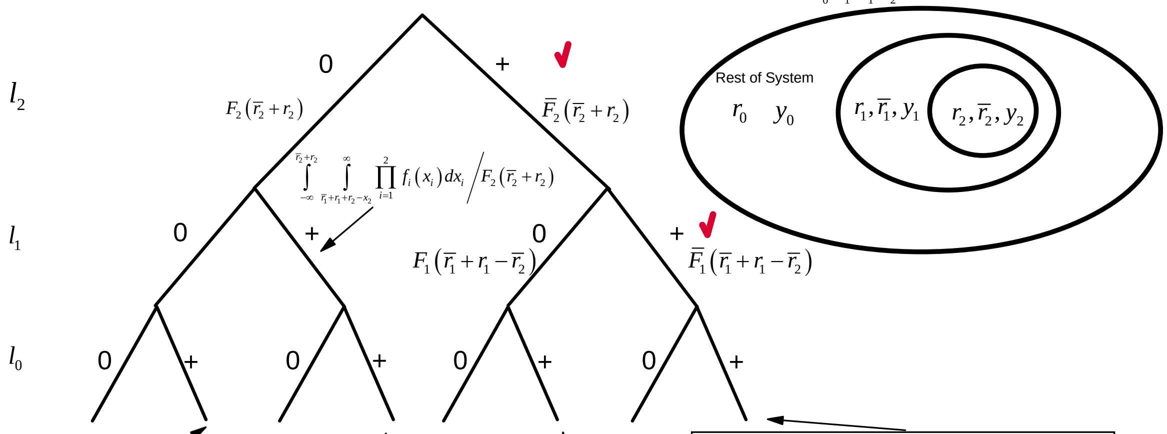
$$\int_{-\infty}^{\bar r_2+r_2}\int_{-\infty}^{\bar r_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i$$

$$\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\int_{r_0-\bar r_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_0(r_0-\bar r_1)\int_{-\infty}^{\bar r_2+r_2}\int_{\bar r_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i$$

$$\int_{\bar r_2+r_2}^{\infty}\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_2(\bar r_2+r_2)\int_{-\infty}^{\bar r_1+r_1-\bar r_2}\int_{r_0+r_1-x_1-\bar r_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$

$$\int_{\bar r_2+r_2}^{\infty}\int_{\bar r_1+r_1-\bar r_2}^{\infty}\int_{r_0-\bar r_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar F_0(r_0-\bar r_1)\bar F_1(\bar r_1+r_1-\bar r_2)\bar F_2(\bar r_2+r_2)$$

The nested loss outcomes and dependencies determine the demand for interface 2 capacity.



$$p_{\bar{r}_2} = v_2\bar{F}_2(\bar{r}_2+r_2) - v_1\bar{F}_1(\bar{r}_1+r_1-\bar{r}_2)\bar{F}_2(\bar{r}_2+r_2) - v_0\bar{F}_2(\bar{r}_2+r_2)\int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2}\int_{r_0+r_1-x_1-\bar{r}_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$

$$\int_{-\infty}^{\bar{r}_2+r_2}\int_{\bar{r}_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i \Big/ F_2(\bar{r}_2+r_2)$$

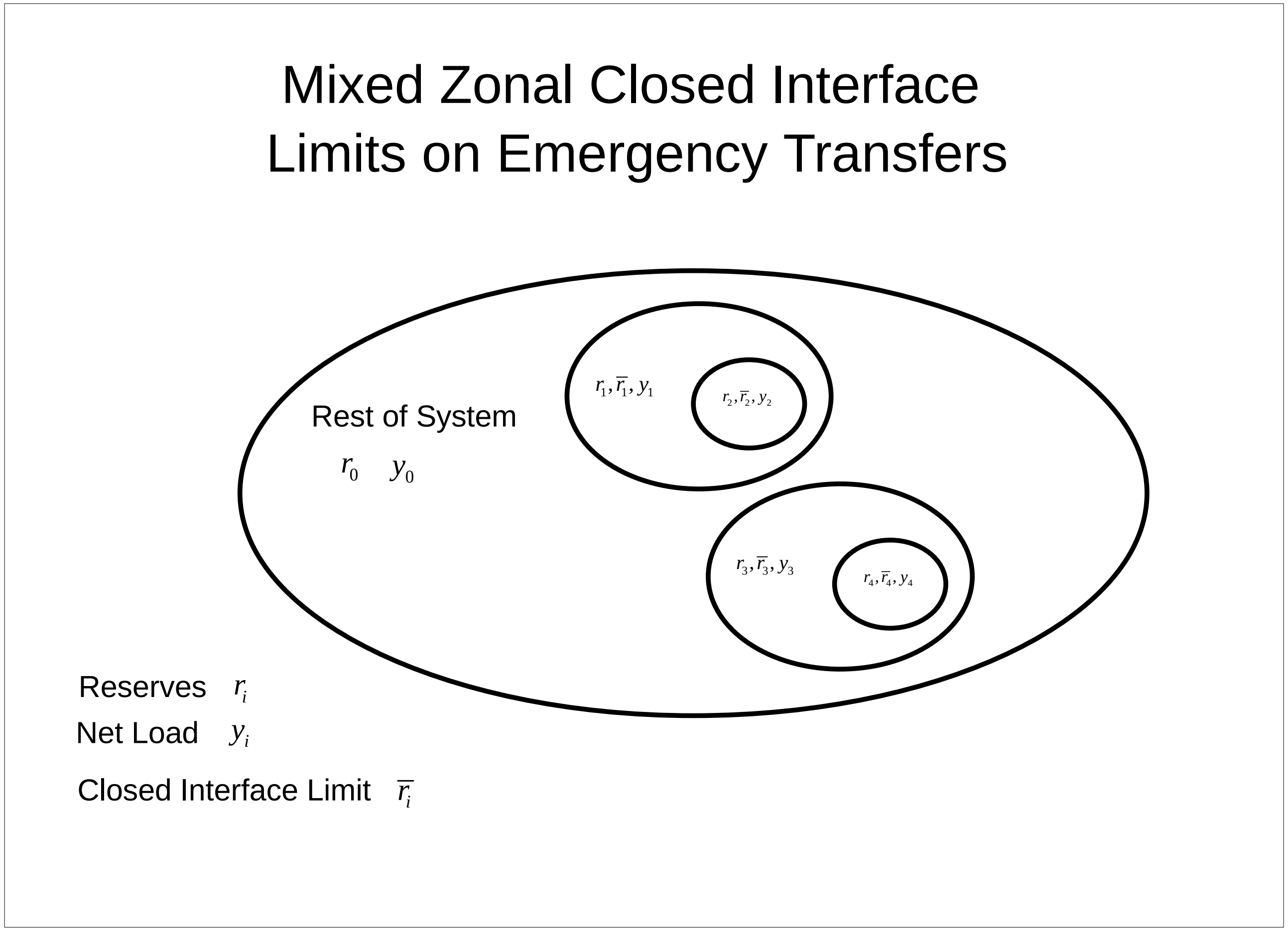
$$\int_{-\infty}^{\bar{r}_2+r_2}\int_{-\infty}^{\bar{r}_1+r_1+r_2-x_2}\int_{r_0+r_1+r_2-x_1-x_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i$$

$$\int_{-\infty}^{\bar{r}_2+r_2}\int_{\bar{r}_1+r_1+r_2-x_2}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar{F}_0(r_0-\bar{r}_1)\int_{-\infty}^{\bar{r}_2+r_2}\int_{\bar{r}_1+r_1+r_2-x_2}^{\infty}\prod_{i=1}^{2} f_i(x_i)\,dx_i$$

$$\int_{\bar{r}_2+r_2}^{\infty}\int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2}\int_{r_0+r_1-x_1-\bar{r}_2}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar{F}_2(\bar{r}_2+r_2)\int_{-\infty}^{\bar{r}_1+r_1-\bar{r}_2}\int_{r_0+r_1-x_1-\bar{r}_2}^{\infty}\prod_{i=0}^{1} f_i(x_i)\,dx_i$$

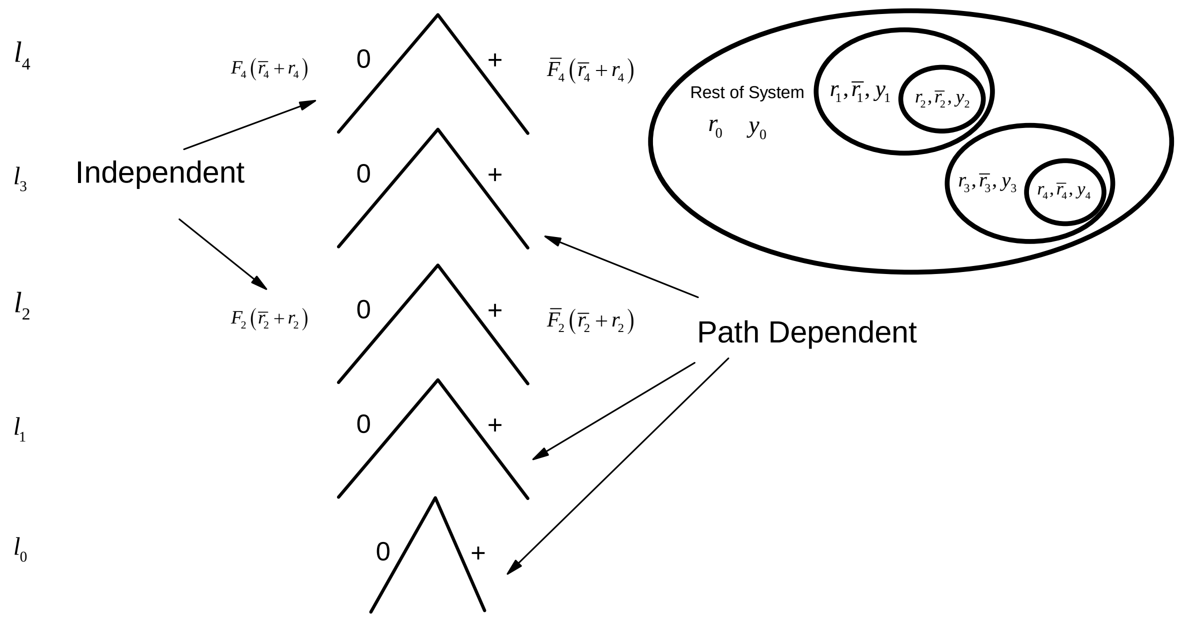
$$\int_{\bar{r}_2+r_2}^{\infty}\int_{\bar{r}_1+r_1-\bar{r}_2}^{\infty}\int_{r_0-\bar{r}_1}^{\infty}\prod_{i=0}^{2} f_i(x_i)\,dx_i = \bar{F}_0(r_0-\bar{r}_1)\bar{F}_1(\bar{r}_1+r_1-\bar{r}_2)\bar{F}_2(\bar{r}_2+r_2)$$

Mixed constrained zones define a more general extension of a constrained zonal structure.



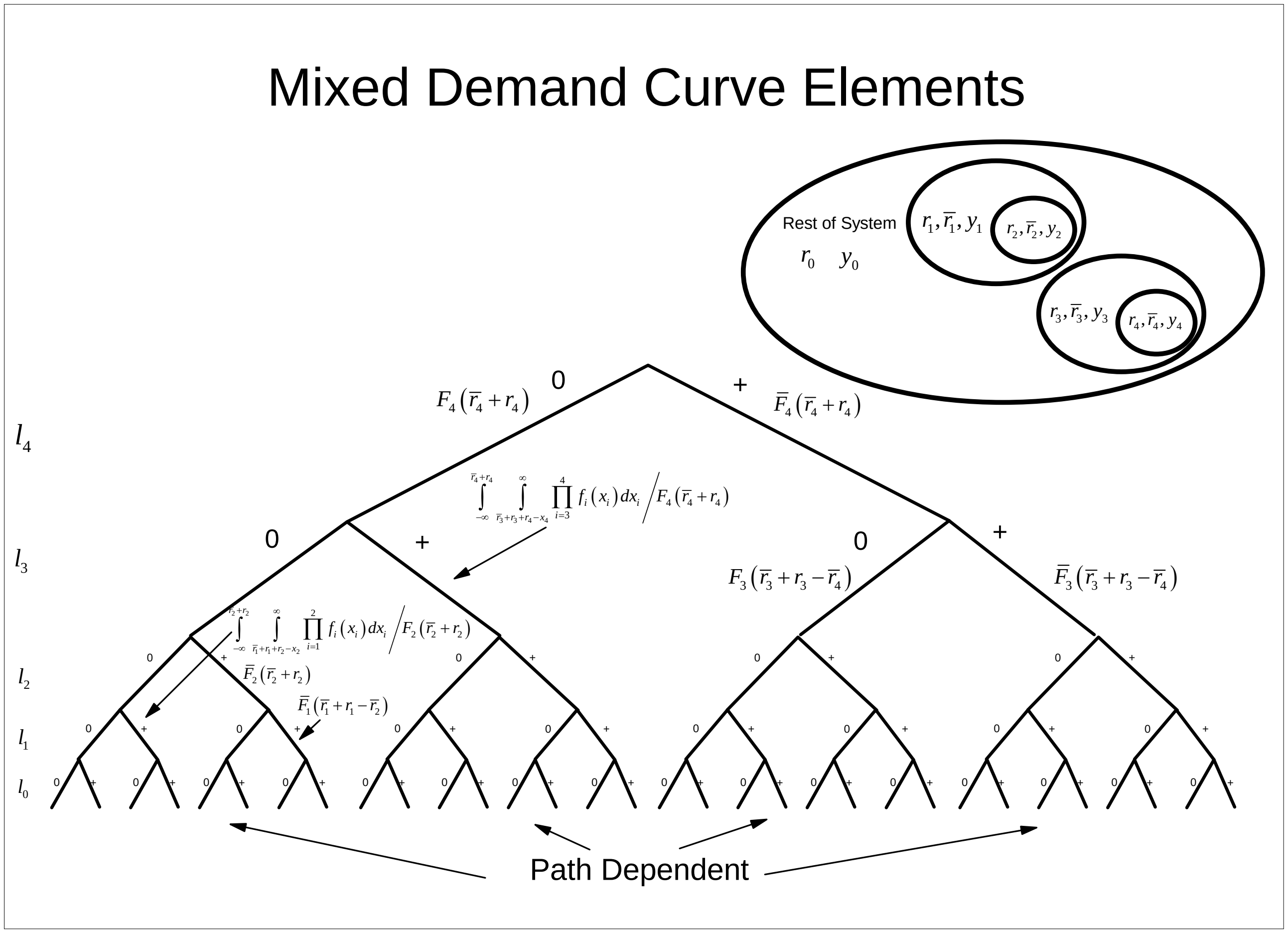
The mixed probability tree for the nested zones captures the dependencies of loss of load.

Mixed Loss of Load Probability Structure

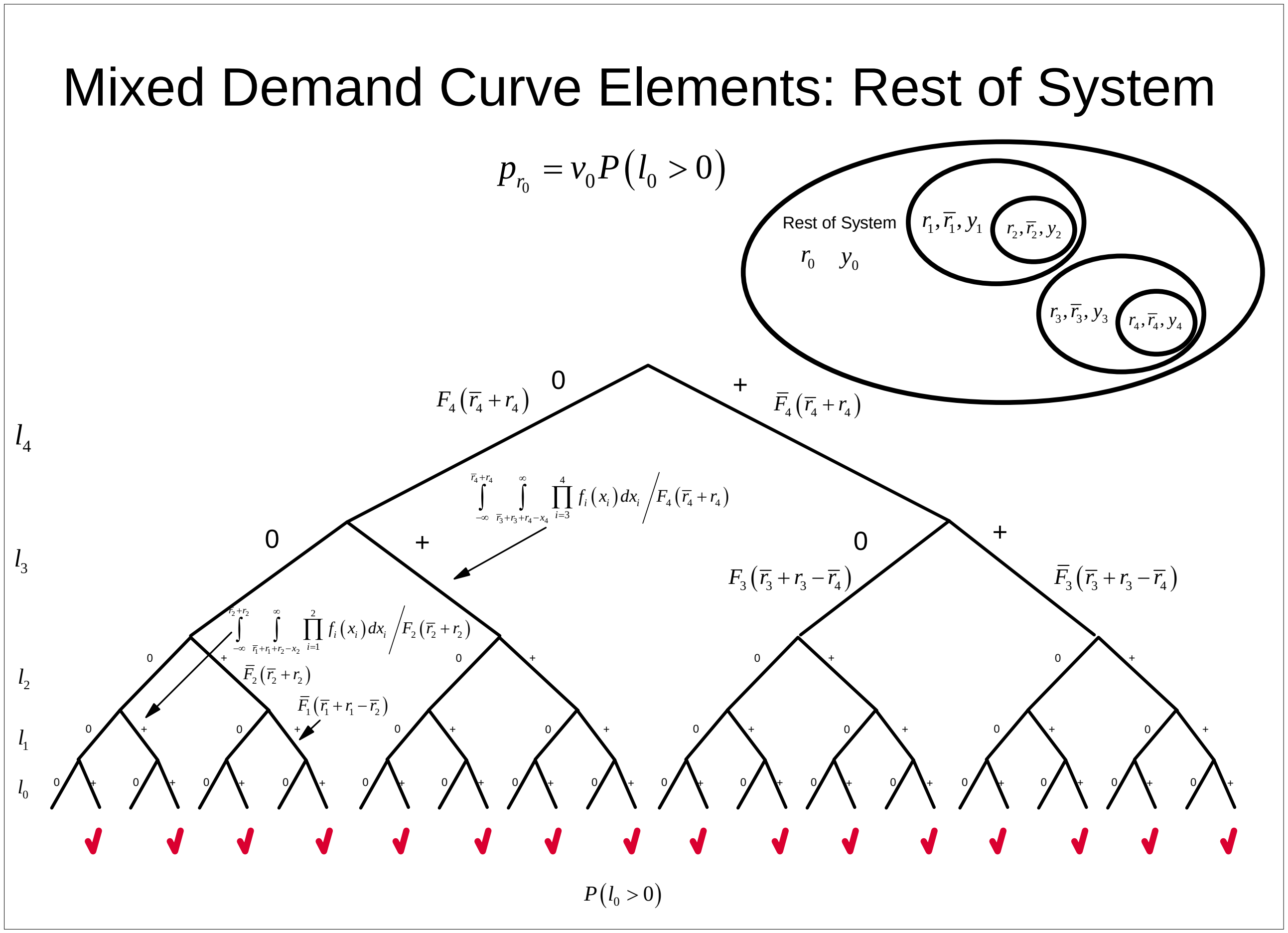


$$y_i \sim f_i,\ F_i(y_i) = \int_{-\infty}^{y_i} f_i(x_i)\,dx_i$$

The mixed loss of load probability structure defines the demand curve elements.



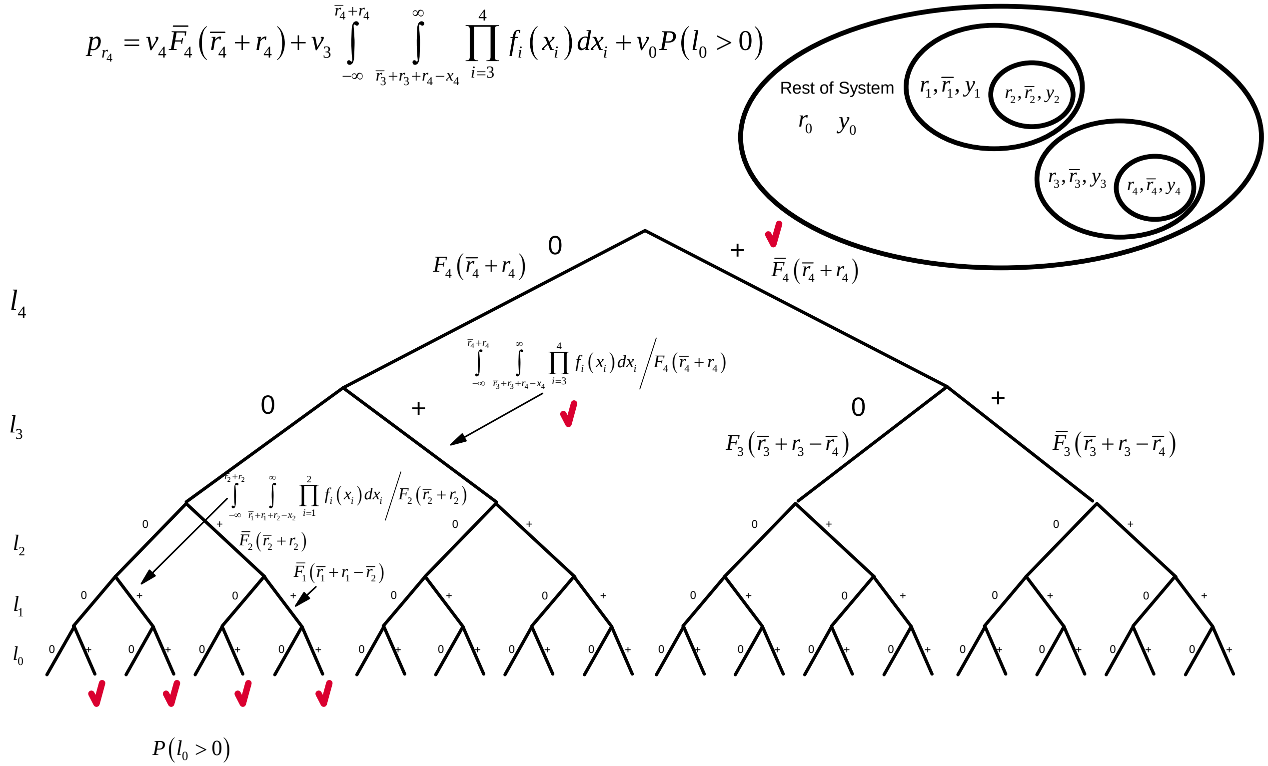
The mixed loss outcomes and dependencies determine the demand for rest of system operating reserves.



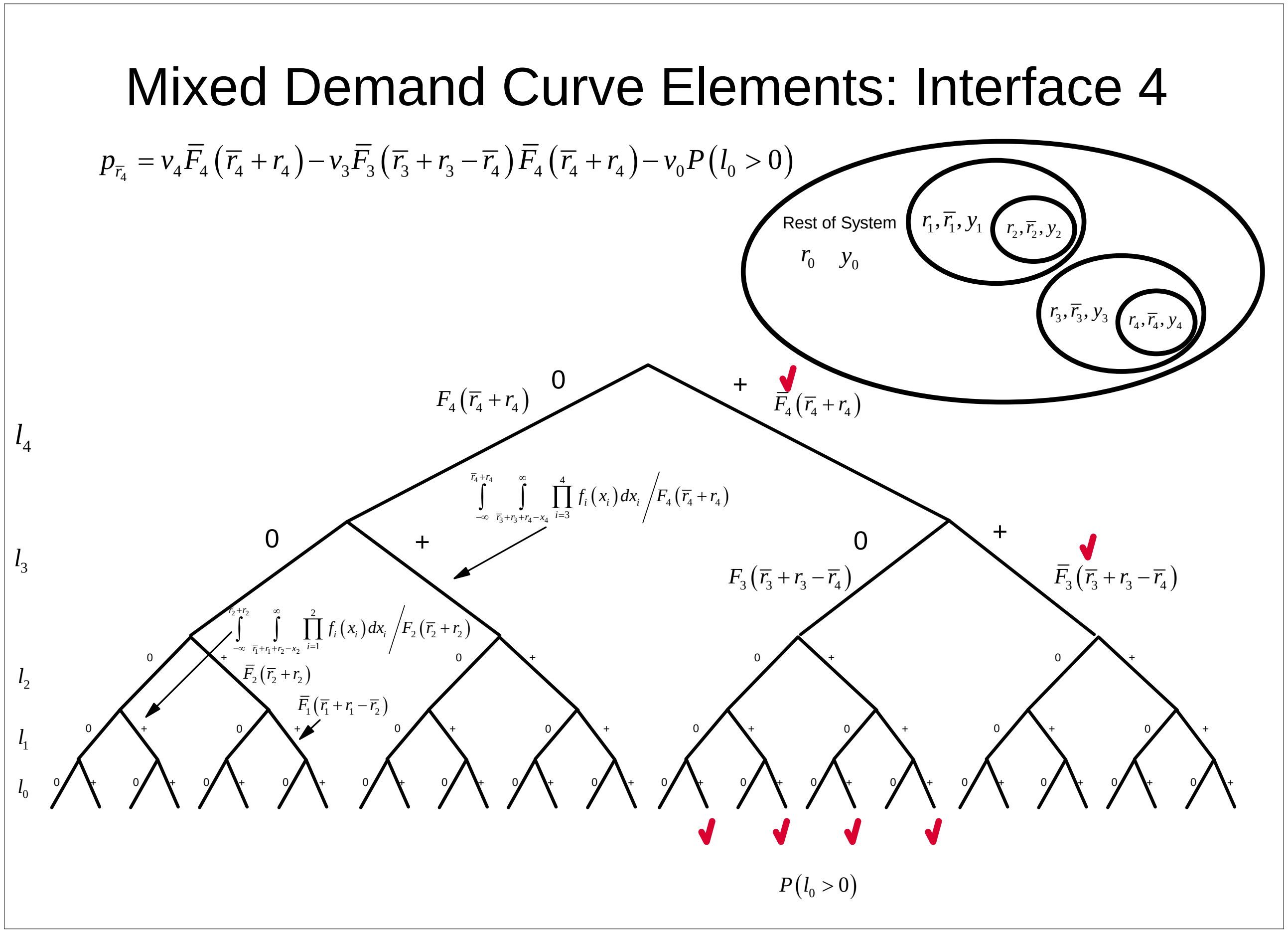
The mixed loss outcomes and dependencies determine the demand for zone 4 operating reserves.

Mixed Demand Curve Elements: Zone 4

$$p_{r_4} = v_4 \overline{F}_4\left(\overline{r}_4 + r_4\right) + v_3 \int_{-\infty}^{\overline{r}_4 + r_4} \int_{\overline{r}_3 + r_3 + r_4 - x_4}^{\infty} \prod_{i=3}^{4} f_i\left(x_i\right) dx_i + v_0 P\left(l_0 > 0\right)$$



The mixed loss outcomes and dependencies determine the demand for interface 4 capacity.



William W. Hogan is the Raymond Plank Professor of Global Energy Policy, John F. Kennedy School of Government, Harvard University and a Director of LECG, LLC. This paper draws on work for the Harvard Electricity Policy Group and the Harvard-Japan Project on Energy and the Environment. The author is or has been a consultant on electric market reform and transmission issues for Allegheny Electric Global Market, American Electric Power, American National Power, Australian Gas Light Company, Avista Energy, Barclays, Brazil Power Exchange Administrator (ASMAE), British National Grid Company, California Independent Energy Producers Association, California Independent System Operator, Calpine Corporation, Canadian Imperial Bank of Commerce, Centerpoint Energy, Central Maine Power Company, Chubu Electric Power Company, Citigroup, Comision Reguladora De Energia (CRE, Mexico), Commonwealth Edison Company, Conectiv, Constellation Power Source, Coral Power, Credit First Suisse Boston, DC Energy, Detroit Edison Company, Deutsche Bank, Duquesne Light Company, Dynegy, Edison Electric Institute, Edison Mission Energy, Electricity Corporation of New Zealand, Electric Power Supply Association, El Paso Electric, GPU Inc. (and the Supporting Companies of PJM), Exelon, GPU PowerNet Pty Ltd., GWF Energy, Independent Energy Producers Assn, ISO New England, Luz del Sur, Maine Public Advocate, Maine Public Utilities Commission, Merrill Lynch, Midwest ISO, Mirant Corporation, JP Morgan, Morgan Stanley Capital Group, National Independent Energy Producers, New England Power Company, New York Independent System Operator, New York Power Pool, New York Utilities Collaborative, Niagara Mohawk Corporation, NRG Energy, Inc., Ontario IMO, Pepco, Pinpoint Power, PJM Office of Interconnection, PPL Corporation, Public Service Electric & Gas Company, PSEG Companies, Reliant Energy, Rhode Island Public Utilities Commission, San Diego Gas & Electric Corporation, Sempra Energy, SPP, Texas Genco, Texas Utilities Co, Tokyo Electric Power Company, Toronto Dominion Bank, TransÉnergie, Transpower of New Zealand, Westbrook Power, Western Power Trading Forum, Williams Energy Group, and Wisconsin Electric Power Company. The views presented here are not necessarily attributable to any of those mentioned, and any remaining errors are solely the responsibility of the author. (Related papers can be found on the web at www.whogan.com).