



# Integer Relaxation for Electricity Market Pricing

Harvard Electricity Policy Group  
January 25-26, 2018

Hung-po Chao, Ph. D.  
Senior Director and Chief Economist  
PJM Interconnection, LLC

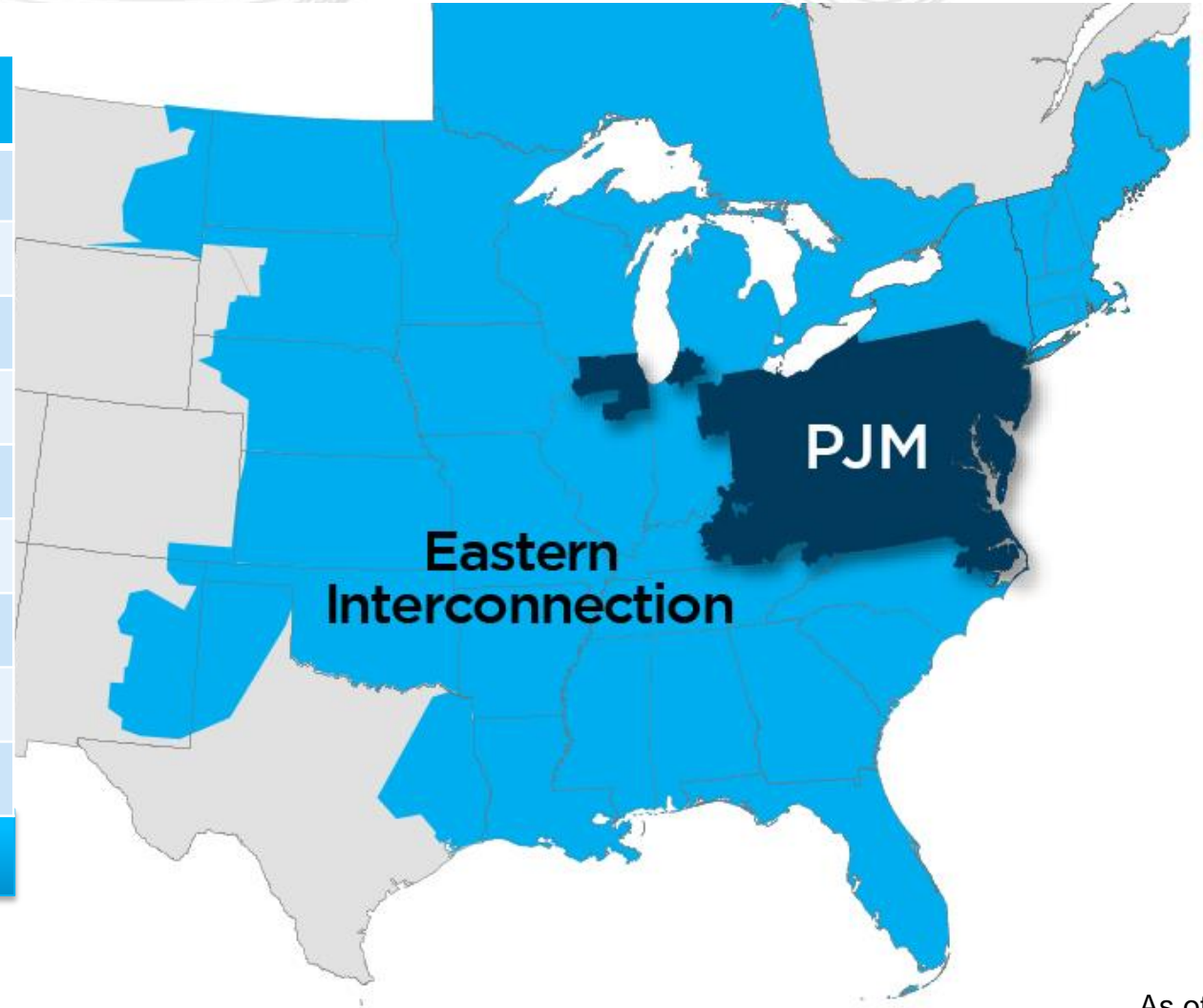


# PJM as Part of the Eastern Interconnection

## Key Statistics

Member companies	1,000+
Millions of people served	65
Peak load in megawatts	165,492
MW of generating capacity	176,569
Miles of transmission lines	82,546
2016 GWh of annual energy	792,314
Generation sources	1,304
Square miles of territory	243,417
States served	13 + DC

**21% of U.S. GDP produced in PJM**



As of 2/2017

- The current LMP pricing method has served the energy market successfully over the past twenty years covering important aspects of efficient pricing
- But there have always been circumstances where the prices could not reflect everything relevant to sending the right market signals
- A growing number of experts recommend that the ELMP and scarcity pricing warrant careful reexamination for enhanced energy and reserve market design
- PJM believes that it is prudent to take an essential first step to improve the foundation of energy pricing to ensure that the prices will more accurately reflect the incremental costs to serve load

- Current LMP does not reflect the true system incremental cost to serve load
  - Participants have incentives to behave inflexibly
- The LMP may be decreasing when demand increases
  - Potential conflict with efficient pricing under shortage conditions
  - Vulnerable to local market power and market manipulation
- Under non-convex conditions, market clearing prices may not exist
  - Units needed to serve load may incur losses and need uplift payments
  - Units not needed may be profitable to run but are required to stay offline

## Why is Extended LMP a good thing?

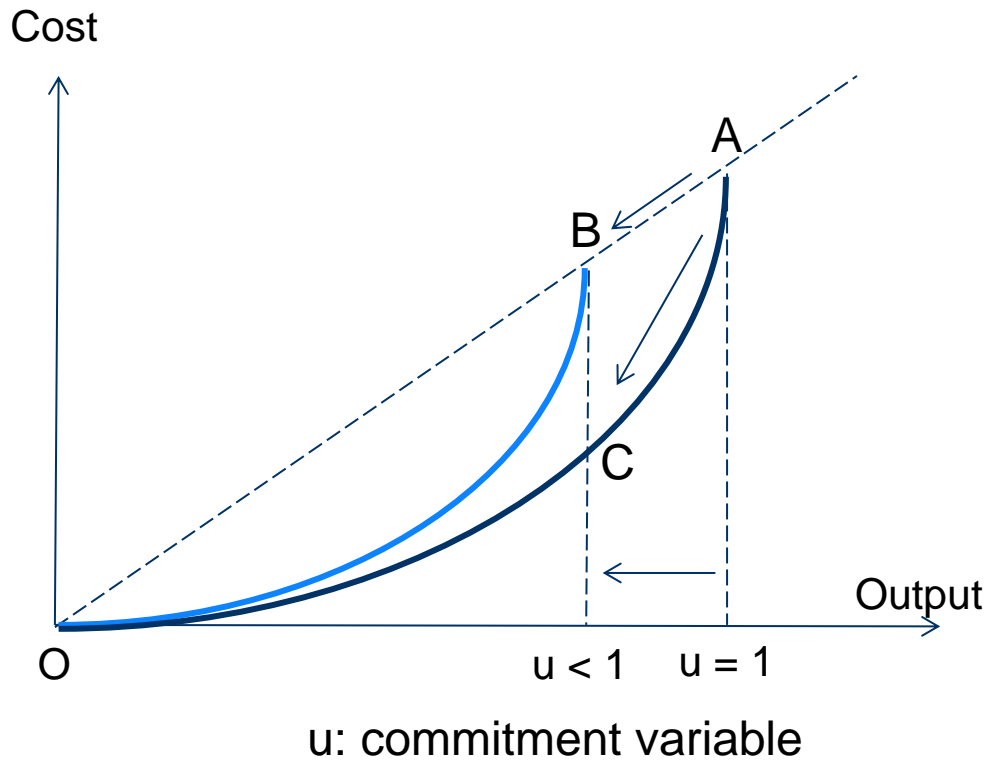
- The extended LMP (ELMP) or convex-hull pricing incorporates non-convex costs of unit commitment (start-up and no-load costs) in market prices with minimum uplift
  - It solves problems caused by non-convexity and fixes pitfalls in the current LMP pricing method
  - It achieves minimum total uplift payment
  - It enables a stronger “invisible hand” sending better market signals
- Issues
  - It bifurcates the dispatch run and the pricing run
  - It creates a computationally challenging problem

- Integer relaxation is a natural approximation to convex-hull relaxation
- The commitment variables must be (0,1) integer values in the dispatch model but in the pricing model, they are allowed to take on any value in the  $[0,1]$  interval
  - Except for different treatments of commitment constraints, the pricing model and the dispatch model are otherwise the same
- The integer relaxation generally provides a good approximation to the convex-hull relaxation
  - It is easier to compute and interpret in implementation
  - MISO has implemented one version of this approach, called Approximated ELMP, for fast-start pricing

- Mixed integer programming (MIP) is now an indispensable tool in business and engineering modeling with a wealth of *formulation* techniques that facilitate integer relaxation
- The unit commitment and economic dispatch (UCED) problem can be reformulated in such a way that the pricing model with integer relaxation solves the convex-hull pricing problem precisely
- Key property: The cost function is positive homogeneous of degree one in both commitment and dispatch decision variables

# A Linear Homogeneous Formulation

- The reformulated cost function changes linearly when the commitment and dispatch variables vary proportionally
- It preserves the same dispatch model but creates a new pricing model



**A to B:** Integer relaxation with homogeneous formulation

**A to C:** Integer relaxation with non-homogenous formulation



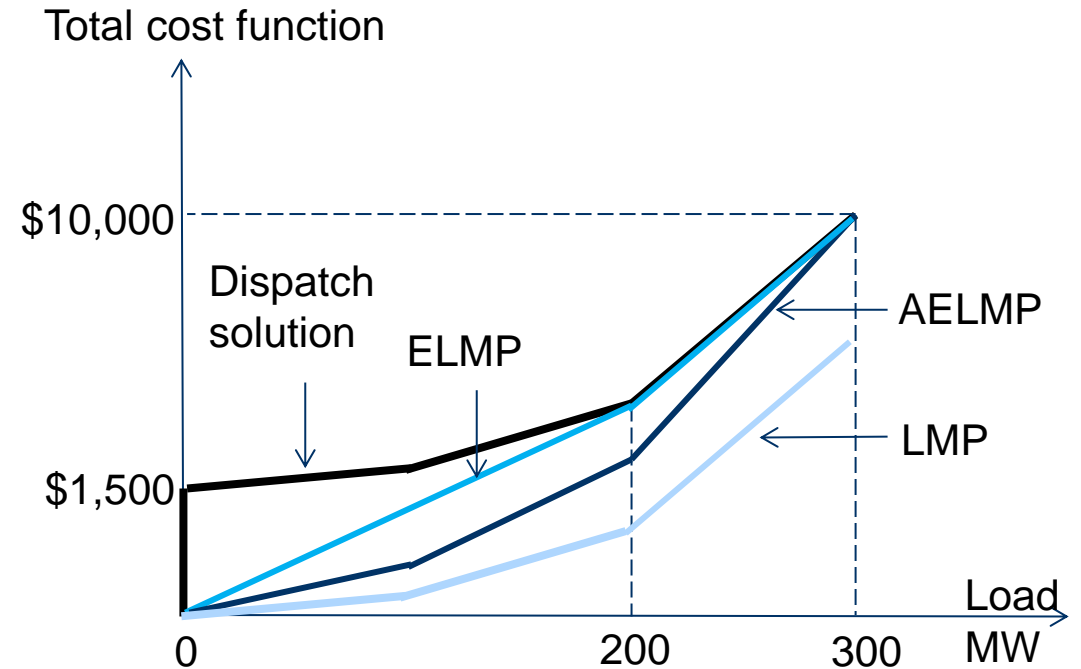
## Cost assumption

Increment (MW)	Energy Cost (\$/MWh)
100	20
100	25
100	40
Fixed cost	\$1,500

## Pricing solution (\$/MWh)

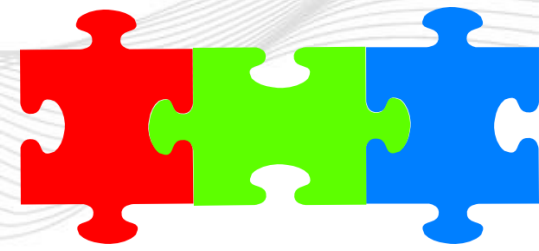
Load (MW)	LMP	AELMP	ELMP
0 – 100	\$20	\$25	\$30
100 - 200	\$25	\$30	\$30
200 - 300	\$40	\$45	\$40

ELMP: IR with linear homogenous formulation  
 AELMP: IR with non-homogenous formulation



- The homogeneous formulation is not restrictive and can cover a broad range of issues such as ramping constraints
  - It is well connected with the theory of peak load pricing
- Integer relaxation expedites the recovery of commitment costs through markets instead of administrative methods
- When ramping constraints are not binding,
  - No-load costs is amortized within each pricing period
  - The start-up cost is amortized in the peak hours
- With ramping constraints, LMPs are adjusted over dispatch periods producing consistent load-following incentives

# Qualitative Assessment of Alternative LMP Pricing Methods

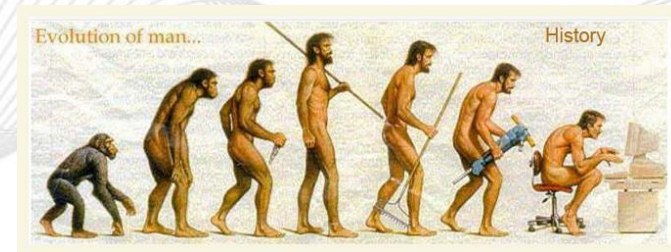


Design Criteria	Restricted LMP Method (Current Method)	Extended LMP Method (Integer relaxation – Proposed Method)	Extended LMP Method (Convex hull relaxation)
Efficient commitment and dispatch	High	High	High
Solutions supported by prices and settlements	Medium	High	High
Incentive-compatible conditions	Low	High	High
Minimized uplift payments	Low	Medium	High
Computationally feasible	High	Medium	Low

## Better market signals and a more effective Invisible Hand

- Allowing all resources needed to serve load to set price while competing for infra-marginal rents
- Extending LMP to include commitment costs
- Improved market transparency with reduced uplift and revenue shifts caused by artificially low energy prices
- Improved price-load relationship consistent with scarcity pricing
- Improved reserve prices more reflective of the reliability value
- Improved performance incentives, especially during tight system conditions
- Improved participative incentives for demand resources and price sensitive demand
- Improved market design with increased information and incentive efficiency to advance social welfare and policy objectives

# What's the next challenge in market evolution?

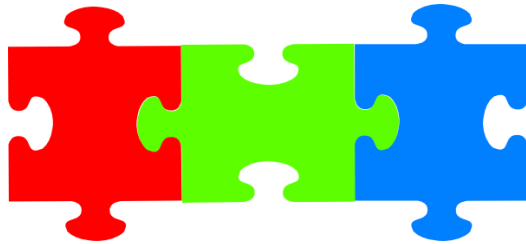


Year	Event
1965	New York Blackout
1978	Public Utility Regulatory Policy Act
1997	FERC Orders 888, 889
2000- 2001	California Electricity Crisis and Enron Market Manipulation
2005	Energy Policy Act with demand response mandate
2011	FERC issued Order 745 on Demand Response Compensation
2014-2015	Order 745 was vacated by the Appeal Court but reversed by the U.S. Supreme Court
2017	DOE issued NOPR on Grid Resilience and Price Formation

# Thank You

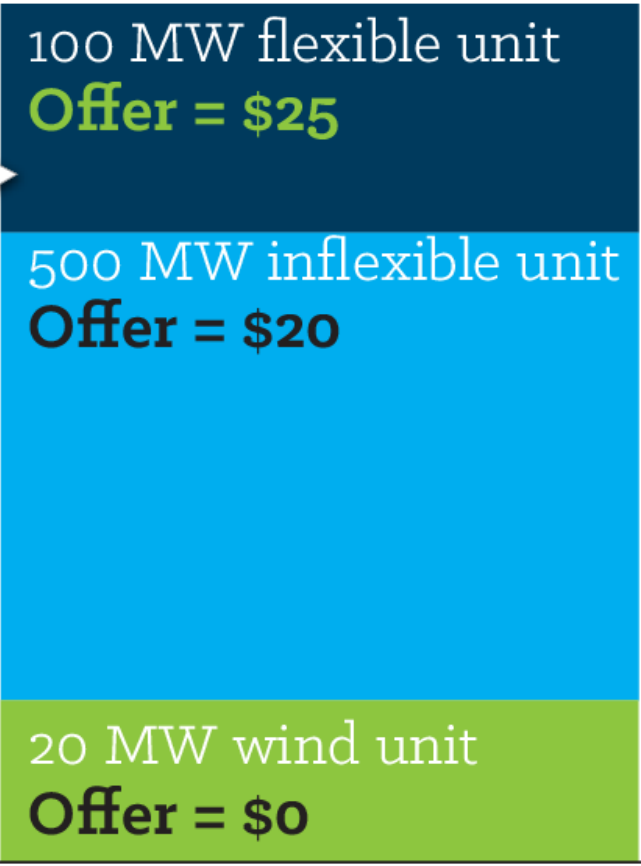


# Appendix



# Current LMP May Not Reflect the True Cost to Serve Load

Load = 530 MW  
**LMP = \$25**



Flexible unit offer:  $\$20 + \$0.1/\text{MW}$

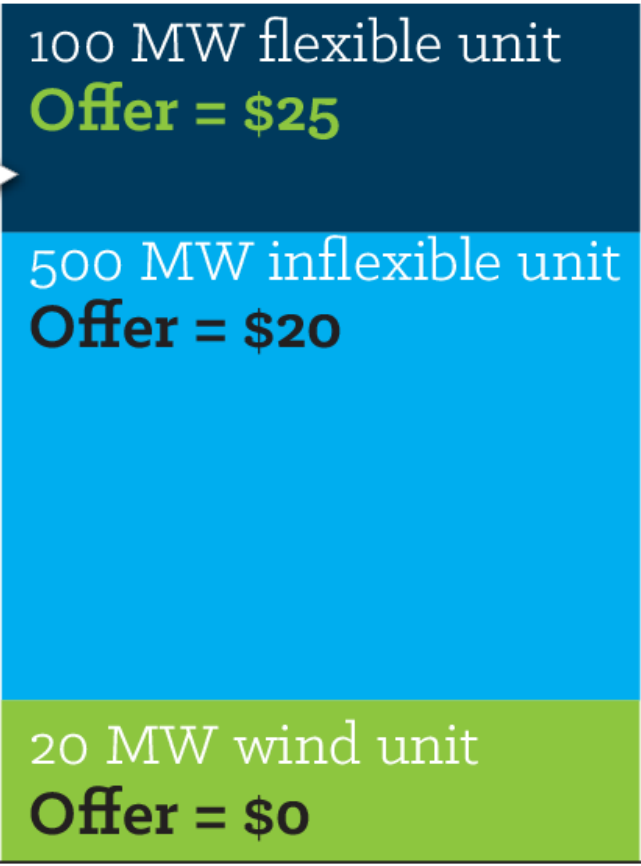
Load = 510 MW  
**LMP = \$0**



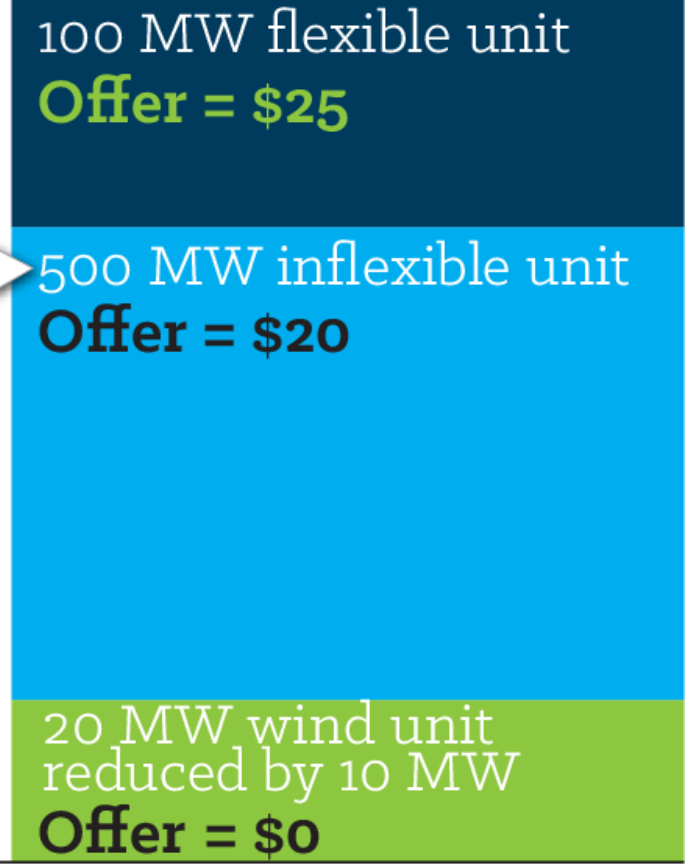


# True Cost to Serve Load Should be More Transparent

Load = 530 MW  
**LMP = \$25**



Load = 510 MW  
**LMP = \$ 20**



# ELMP Supports Efficient Commitment and Dispatch Solution

## Hour 1

LMP = \$35/MWh  
Total cost = 16,500

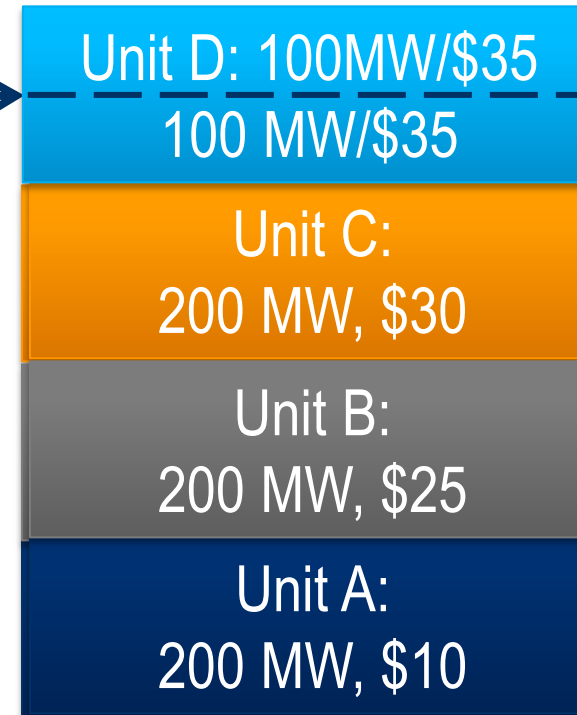
Load =  
700 MW



## Hour 2

LMP = \$35/MWh  
Total bid-cost = 16,500

Load =  
700 MW



Units A and D are flexible

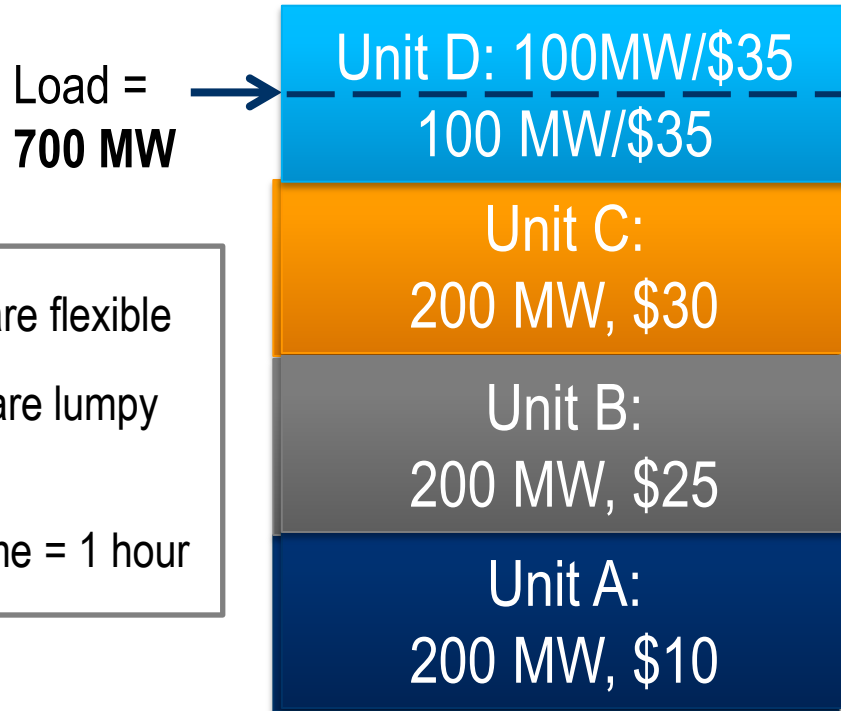
Units B and C are lumpy  
or block-loaded

Minimum runtime = 1  
hour

# Current LMP Pricing Method: Declining Demand

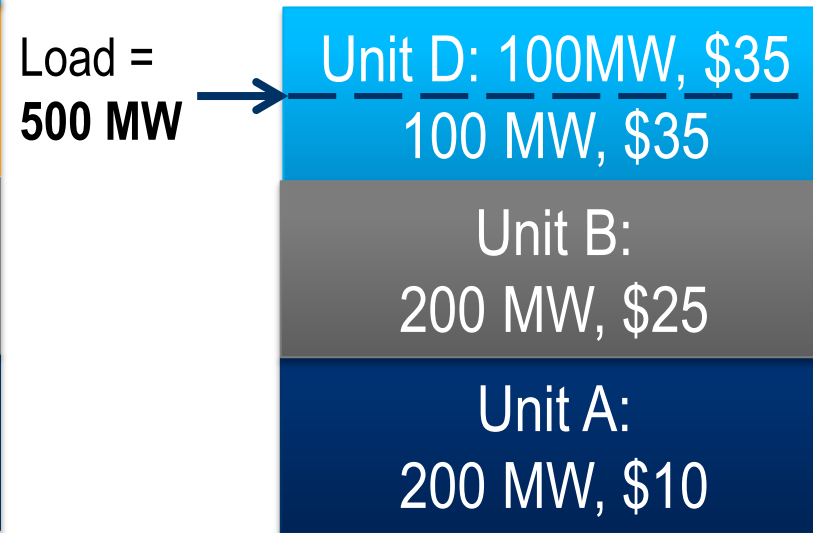
## Hour 1

LMP = \$35/MWh  
 Total cost = 16,500



## Hour 2

LMP = \$35/MWh  
 Total bid-cost = 10,500



Units A and D are flexible  
 Units B and C are lumpy or block-loaded  
 Minimum runtime = 1 hour

# Current Pricing Rule Incentivizes Unit C to Bid Inflexibly

## Hour 1

LMP = \$35/MWh  
Total cost = 16,500

## Hour 2

LMP = \$35/MWh  
Total bid-cost = 11,500

Load =  
700 MW



Load =  
500 MW



Unit B:  
200 MW, \$25

Unit C raises  
Minimum Runtime  
to 2 hours and  
replaces Unit B

Units A and D are flexible

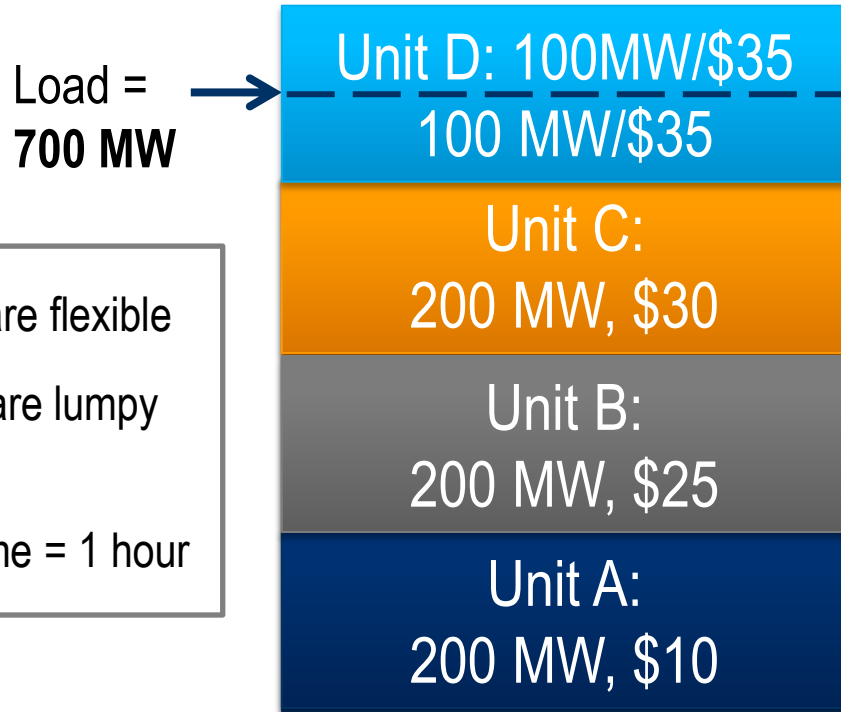
Units B and C are lumpy  
or block-loaded

Minimum runtime = 1 hour

# Current Pricing Rule Incentivizes Units B and C to Bid Inflexibly

## Hour 1

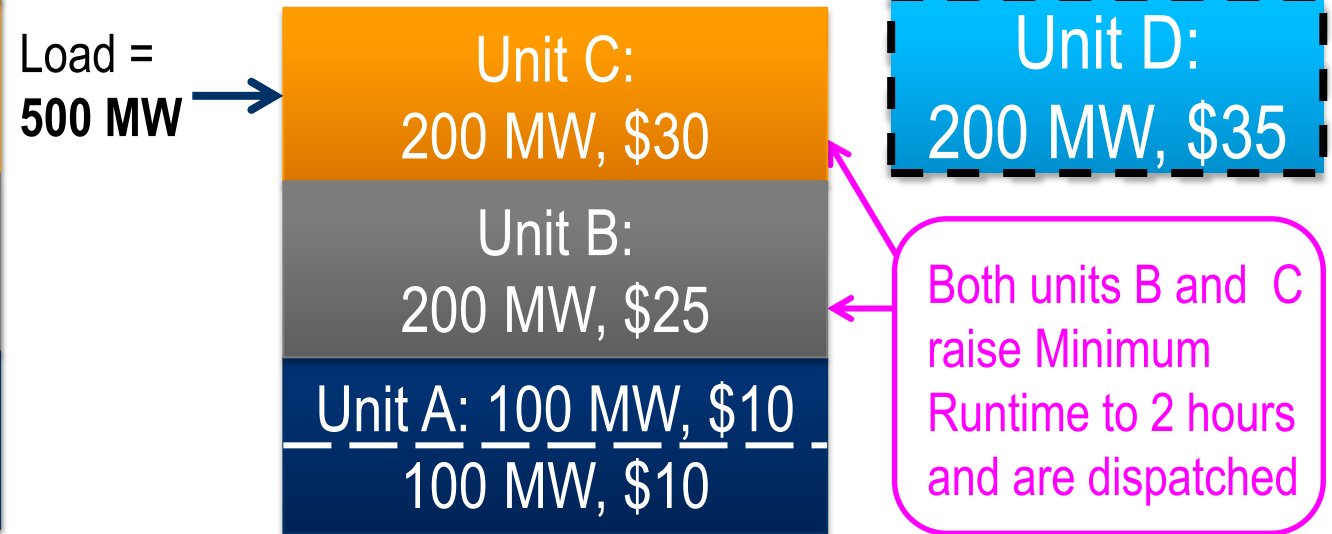
LMP = \$35/MWh  
Total cost = 16,500



Units A and D are flexible  
Units B and C are lumpy or block-loaded  
Minimum runtime = 1 hour

## Hour 2

LMP = \$10/MWh  
Total bid-cost = 12,000  
Uplift = \$4,000

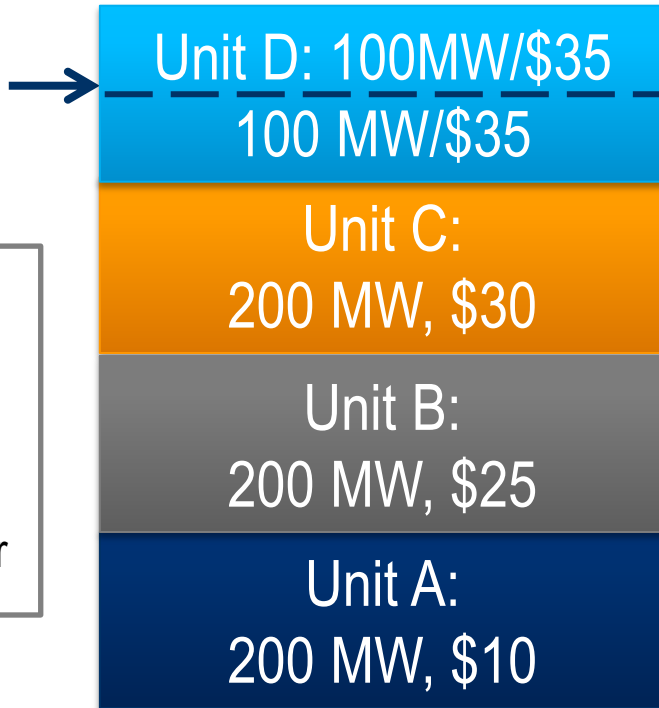


# Extended LMP Solution Supports Efficient Dispatch with Reduced Uplift

## Hour 1

LMP = \$35/MWh  
Total cost = 16,500

Load =  
**700 MW**

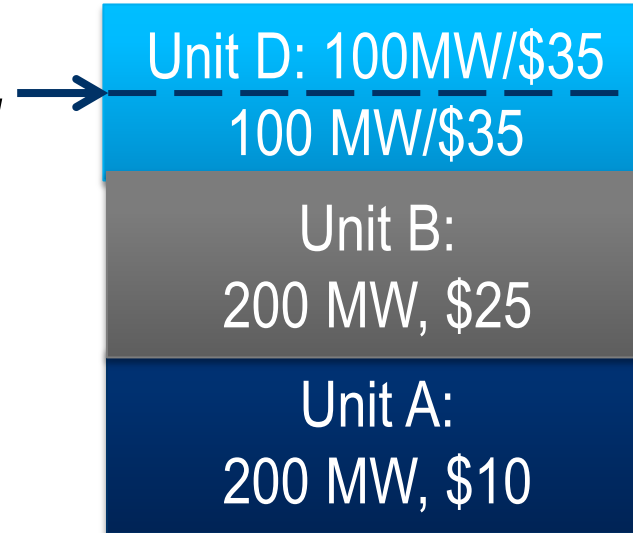


Units A and D are flexible  
Units B and C are lumpy or block-loaded  
Minimum runtime = 1 hour

## Hour 2

LMP = \$35/MWh  
Total cost = 10,500  
Uplift for Unit C = \$1,000

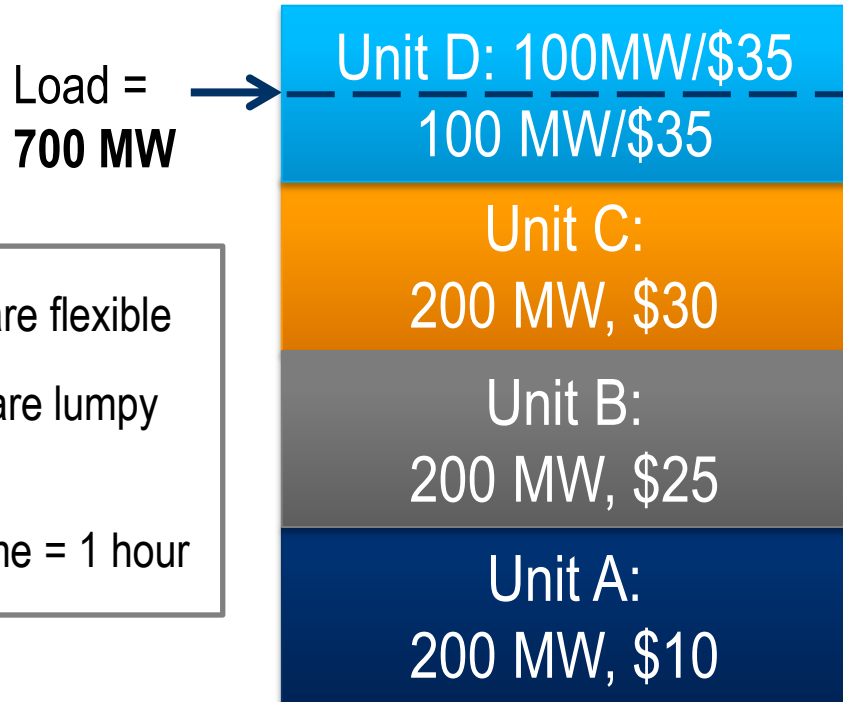
Load =  
**500 MW**



# Extended LMP Solution Supports Efficient Dispatch with Minimum Uplift

## Hour 1

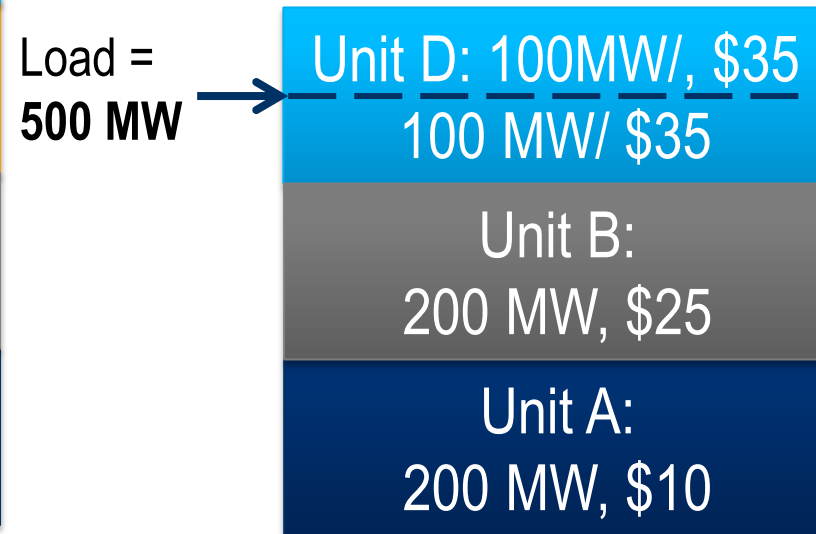
LMP = \$35/MWh  
Total cost = 16,500



Units A and D are flexible  
Units B and C are lumpy or block-loaded  
Minimum runtime = 1 hour

## Hour 2

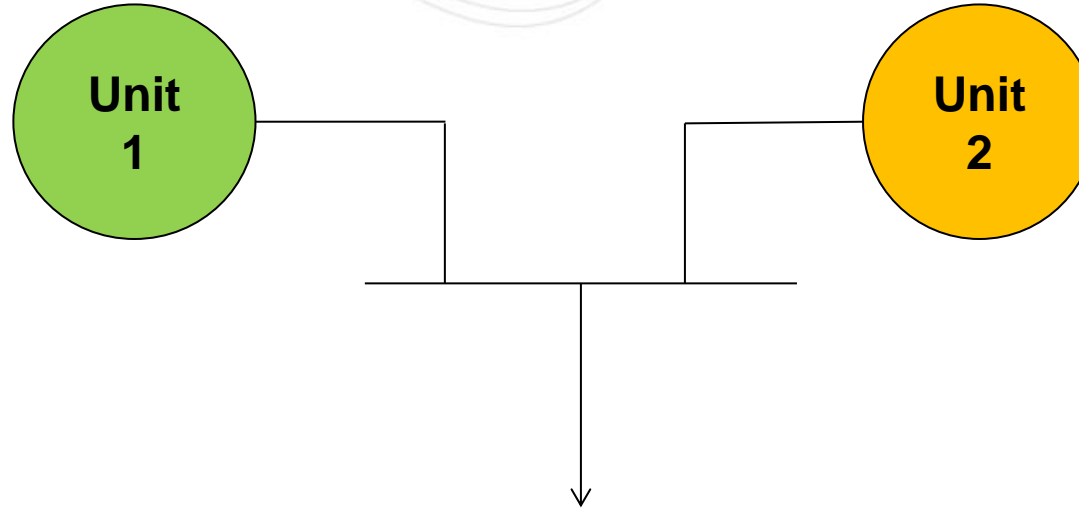
LMP = \$30/MWh  
Total bid-cost = 10,500  
Uplift for Unit C = \$500



# Example 2

## 1 Bus, 2 Units, 3 Periods

Parameter	Value
Energy cost	\$40/MWh
Start-up cost	\$500/start
No-load cost	\$1000/hour
Max output	100 MW
Eco Min	40 MW
Ramp rate	100 MW/hour



Hour	Demand (MW)
1	$80 - 0.1 \times \text{Price}$
2	$100 - 0.1 \times \text{Price}$
3	$150 - 0.1 \times \text{Price}$

Parameter	Value
Energy cost	\$20/MWh
Start-up cost	\$0/start
No-load cost	\$0/hour
Max output	50 MW
Eco Min	0 MW
Ramp rate	50 MW/hour



Hour	Unit 1 (MW)	Unit 2 (MW)	Load (MW)	LMP (\$/MWh)
1	40	38	78	20
2	46	50	96	40
3	96	50	146	40

- Unit 1 and Unit 2 are both “marginal” units
- The LMP in Hour 1 is below the marginal cost of Unit 1
- LMPs do not reflect commitment costs

Hour	Unit 1 (MW)	Unit 2 (MW)	Load (MW)	LMP (\$/MWh)
1	25.0	50.0	75.0	50.0
2	45.0	50.0	95.0	50.0
3	94.5	50.0	144.5	55.0

- In this case, ELMP covers no-load costs in each hour
- In this case, ELMP covers the start-up cost in the peak hour
- The pricing dispatch results are not used in settlement

	Dispatch solution			Pricing solution	
	Hour	Unit 1 (MW)	Unit 2 (MW)	Load (MW)	LMP (\$/MWh)
ELMP	1	40	38	78	50
	2	46	50	96	50
	3	96	50	146	55
	<b>Uplift</b>	<b>\$1,200</b>	<b>\$360</b>	<b>\$61</b>	<b>Total: \$1,621</b>

- The uplift payment for Unit 1 covers its losses during the commitment period
- The uplift payments for Unit 2 and Load represent the lost opportunity costs

## ELMP Case 2a

	Ramping rate (MW/hour)
Unit 1	60
Unit 2	30

## Dispatch solution

Hour	Unit 1 (MW)	Unit 2 (MW)	Load (MW)	LMP (\$/MWh)
1	46	30	76	40
2	46	50	96	40
3	96	50	146	40

## Pricing solution

Hour	Unit 1 (MW)	Unit 2 (MW)	Load (MW)	LMP (\$/MWh)
1	43.5	30	73.5	65
2	45.5	50	95.5	45
3	94.5	50	144.5	55
<b>Uplift</b>	<b>\$680</b>	<b>\$0</b>	<b>\$44</b>	<b>Total: \$724</b>