Nodal Pricing for Distribution Networks: Efficient Pricing for Efficiency Enhancing Distributed Generation

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***Abstract*— As distributed generation (DG) becomes more widely deployed distribution networks become more active and take on many of the same characteristics as transmission. We propose the use of nodal pricing that is often used in the pricing of short-term operations in transmission. As an economically efficient mechanism, nodal pricing would properly reward DG for reducing line losses through increased revenues at nodal prices, and signal prospective DG where it ought to connect with the distribution network. Applying nodal pricing to a model distribution network we show significant price differences between busses reflecting high marginal losses. Moreover, we show the contribution of a DG resource located at the end of the network to significant reductions in losses and line loading. We also show the DG resource has significantly greater revenue under nodal pricing reflecting its contribution to reduced line losses and loading.**

***Index Terms*— Distribution Networks, Distributed Generation, Nodal Pricing, Loss Allocations.**

I. Introduction

As distributed generation (DG) becomes more widely deployed in distribution networks, distribution networks take on many of the same characteristics as transmission in that they become more active rather than passive. Consequently, pricing mechanisms that have been employed in transmission networks are good candidates for use in distribution networks. One such candidate is nodal pricing, or as it is known to many industry professionals, locational marginal pricing (LMP). Nodal pricing first proposed and developed in [1] provides a pricing mechanism for short term operation of transmission systems that is economically efficient. To date nodal pricing, or a close variant, has been adopted by electricity markets in New York, New England, PJM, New Zealand, Argentina, and Chile. Moreover, other markets in the United States such as those in California and Texas are proposing to use nodal pricing going forward. Clearly, this is a pricing mechanism with which there is a great deal of experience and confidence.

Some may question the use of nodal pricing at the distribution level. Afterall, nodal pricing is most often associated with congestion as discussed in [2], and distribution systems are designed so as to avoid congestion to ensure the obligation to serve is not endangered. However, another aspect to nodal pricing is the pricing of line losses at the margin. In addition to operating at lower voltages with smaller conductors, many distribution networks have areas with low customer densities and long lines. Taken together, marginal losses in distribution can be substantial in many systems over distances that are quite short relative to transmission.

In this paper we propose using nodal pricing in distribution networks to send the right price signals to locate DG resources, and to properly reward DG resources for reducing line losses through increased revenues derived from prices that reflect marginal costs. We apply nodal pricing to a model distribution network with long lines that would be typical for a low customer density network such as in rural areas that is representative of Uruguay. We show significant price differences between busses as well as the significant contribution to the reduction of losses and line loading from a DG resource located at the end of the network. We also show under nodal pricing the DG resource has significantly greater revenue reflecting its contribution to reduced line losses and loading.

The paper is structured as follows. In Section II we will derive the nodal prices for the distribution network. In Section III we will show the difference in revenue for DG resources under nodal pricing versus receiving the price at the PSP. In Section IV we will present an application of the proposed method considering a rural radial distribution network. Section V offers conclusions and directions for future work.

II. Nodal Pricing in a Distribution Network

The manner in which we derive nodal factor prices in a distribution network is no different from deriving them for an entire power system. Let t be the index of time. Let generators at each bus be indexed by g, loads at each bus be indexed by d and busses be indexed k. Define P_{kg}, Q_{kg} respectively, as the active and reactive power injected by generator g located at bus k. For the purposes of our exposition, the interface between generation and transmission, the power supply point (PSP), is treated as a bus with only a generator. Similarly, define P_{kd}, Q_{kd} respectively, as the active and reactive power consumed by demand d at bus k. P and Q without subscripts represent the active and reactive power matrices respectively.

Let $C_{kg}(P_{kg}, Q_{kg})$ be the total cost of producing active and reactive power by generator g at bus k where C_{k_g} is assumed to be convex, weakly increasing, and once continuously differentiable in both of its arguments.

The optimization problem for dispatching distributed generation and power from the power supply point (PSP) can be represented as the following least-cost dispatch problem at each time t:

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$$\min_{\substack{P_{kgt},Q_{kgt}\\ \forall kg,kd}} \sum_k \sum_g C_{kg}(P_{kgt}, Q_{kgt}) \tag{1}$$

subject to
1) Electric balance:

$$Loss(P,Q) - \sum_k \sum_g P_{kgt} + \sum_k \sum_d P_{kdt} = 0, \forall t \tag{2}$$

2) Prime mover and thermal generators' constraints:

$$\begin{array}{c} 0 \leq P_{kgt} \leq \overline{P}_{kg} \\ P_{kgt}^2 + Q_{kgt}^2 \leq \overline{S}_{kg}^2 \end{array} \quad \forall kgt \tag{3}$$

We assume no network constraints at the distribution level as this would imply, in many distribution networks, the curtailment of load which regulators strongly discourage due to obligations to serve all load.

Moreover, we will consider that $Loss(P,Q)$ is convex, increasing, and once continuously differentiable in all of its arguments. Under these hypothesis, application of the Karush-Kuhn-Tucker conditions lead to a system of equations and inequalities that guarantee the global maximum [3].

Define the net withdrawal position for active and reactive power at each bus k at time t by $P_{kt} = \sum_d P_{kdt} - \sum_g P_{kgt}$ and $Q_{kt} = \sum_d Q_{kdt} - \sum_g Q_{kgt}$. Nodal prices are calculated using power flows locating the "reference bus" at the PSP, so λ_t corresponds to the active power price at the PSP. Assuming interior solutions we obtain the following prices for active and reactive power respectively:

$$pa_{kt} = \lambda_t(1 + \frac{\partial Loss}{\partial P_{kt}}), pr_{kt} = \lambda_t(\frac{\partial Loss}{\partial Q_{kt}}) \tag{4}$$

Implicit in our loss function are equations representing the laws governing power flows. We observe that the partial derivative of the power system losses with respect to the extracted active and reactive power at bus k must be evaluated at the values of the electrical variables that correspond to the steady state equilibrium point for a given optimal dispatch. If V, θ are the state variables in the power flow problem, then the partial derivatives of losses with respect to P and Q can be found applying the standard chain rule solving the system of linear equations:

$$\begin{bmatrix} \frac{\partial Loss}{\partial V} \\ \frac{\partial Loss}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial V} & \frac{\partial Q}{\partial V} \\ \frac{\partial P}{\partial \theta} & \frac{\partial Q}{\partial \theta} \end{bmatrix} \begin{bmatrix} \frac{\partial Loss}{\partial P} \\ \frac{\partial Loss}{\partial Q} \end{bmatrix}$$

III. DG Revenue: Nodal Pricing Versus Price = λ_t

Suppose as the alternative to nodal pricing, the DG resource would receive the price at the interface with the transmission system at each time period, λ_t. Over all time periods during the year, the DG resource would then have revenue equal to

$$R_\lambda = \sum_t \lambda_t P_{kt}. \tag{5}$$

This revenue does not reflect a DG resource's contribution to losses (either positive or negative), as does nodal pricing. The nodal pricing revenue for a DG resource located at bus k over the year is expressed as

$$R_n = \sum_t (\lambda_t(1 + \frac{\partial Loss}{\partial P_{kt}}))P_{kt} + \lambda_t(\frac{\partial Loss}{\partial Q_{kt}})Q_{kt} \tag{6}$$

The difference in revenue between receiving the nodal price and simply receiving λ in each time period is

$$R_n - R_\lambda = \sum_t \lambda_t[\frac{\partial Loss}{\partial P_{kt}} P_{kt} + \frac{\partial Loss}{\partial Q_{kt}} Q_{kt}] \tag{7}$$

The difference in revenue is simply the contribution toward the reduction (increase) in losses. If the DG resource reduces losses, then nodal pricing will yield higher revenue. However, if it increases losses, it will receive less revenue.

IV. An Example

We consider a rural radial distribution network, meant to reflect conditions in Uruguay where there are long lines. The network is shown in Fig. 1. The overhead lines in the network are type 120AlAl with $r(\Omega/km) = 0.3016$ and $x(\Omega/km) = 0.3831$. Bus (1) is fed by a 150/30 kV transformer, and 4 radial feeders (A, B, C, D), but for simplicity, we will just consider feeder A for our calculations. Feeder A consists of a 30 kV overhead line feeding 5 residential 30/15 kV busses (3, 5, 6, 7, 8) and an industrial customer at bus 4.

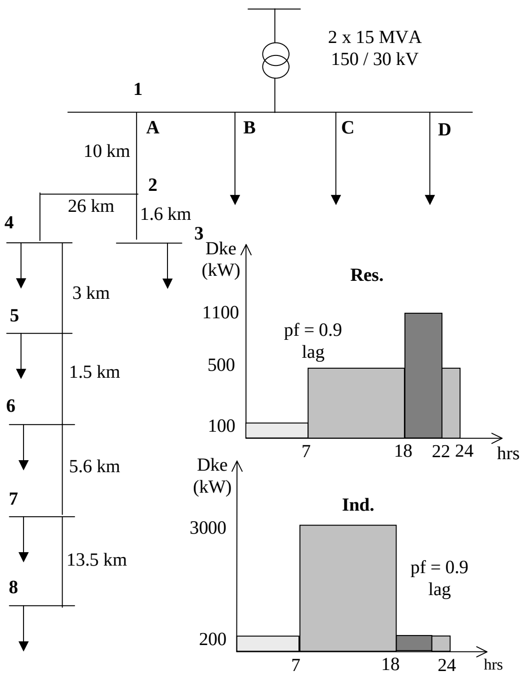

Fig. 1. A rural distribution network with Residential and Industrial Load Profiles

The daily load profiles for the busses are shown in Fig. 1 are also reflective of what might be observed in Uruguay. We will assume then that residential customers have the simplified load profile of Res. in Fig. 1 and the industrial customer the simplified load profile of Ind. in Fig. 1.

There are four different time periods during the day as can be seen in Fig. 1 and for the ease of exposition we do not include seasonal variations. The load periods along with the prices in USD/MWh at bus 1 (PSP) are given in Table I:

TABLE I

LOAD PROFILE AND PRICES

Time	Hours	Load	Price (λ)
Off-Peak (OP)	0 to 7	700 kW	16
Shoulder Day (SD)	7 to 18	5500 kW	24
Peak (P)	18 to 22	5700 kW	30
Shoulder Night (SN)	22 to 24	2700 kW	24

We optimize the network following [4] for two cases: i) no DG resource; and ii) a 1 MVA DG resource located at bus 8 operating at 0.95 lagging power factor, and assuming the DG resource has a cost that is below λ_t in all hours t. A summary of the network impacts is shown in Table II. Prices at bus 8 at the end of the network with and without DG in all time periods are shown in Table III, and prices in the peak period for all busses is shown in Table V. The revenue for DG under nodal pricing and "λ pricing" is shown in Table IV.

TABLE II

SUMMARY NETWORK RESULTS

	No DG	DG	% difference
I_{max}	137.0	112.0	18
$Max\Delta V(\%)$	13.9	10.4	25
$TotalLosses(MWh/yr)$	2946.1	1844.7	37
$TotalLoss(USD/yr)$	75243	46986	37

TABLE III

PRICES AT BUS 8 AND DG REVENUE

	No DG		DG	
Time	p_a	p_r	p_a	p_r
OP	16.2976	0.1456	15.6928	-0.0512
SD	28.8336	2.6496	27.1704	1.9056
P	36.732	3.702	34.473	2.634
SN	25.9872	1.0176	24.8448	0.5832

TABLE IV

DG REVENUE: NODAL VS. λ

R_λ	R_n	% difference
188632	210448	12

TABLE V

PRICES AT ALL BUSSES AT PEAK

	No DG		DG	
Bus	p_a	p_r	p_a	p_r
1	30	0	30	0
3	31.503	0.9	31.182	0.702
4	35.118	2.901	33.771	2.184
5	35.571	3.129	34.083	2.349
6	35.742	3.216	34.191	2.409
7	36.183	3.432	34.41	2.541
8	36.732	3.702	34.473	2.634

V. DISCUSSION AND CONCLUSION

This paper has presented the nodal pricing scheme applied to distribution networks with distributed generation (DG). From Table II it is apparent that a DG resource when properly located (at the end of the network) can provide benefits to the network through reduced line losses and line loading by 37% and 18% respectively as well as reducing voltage changes by 25%. We contend that DG resources should be appropriately rewarded, through nodal pricing, for providing such benefits to the distribution system. From Tables III and V we can see the price impact of losses without the DG resource and then the reduction in prices with the DG resource. Moreover, the revenue obtained by the DG resource is 12% greater under nodal pricing of distribution than if it were simply paid the price at the PSP, λ_t.

Without the efficient incentives presented by nodal pricing through higher prices leading to larger revenues for DG resources, there is not much of hope of inducing DG resources to locate and operate so they can provide the system benefits as shown above. Given worldwide experience with nodal pricing, and the fact that DG resources transform the distribution network into an active network like transmission, it makes sense to consider nodal pricing in distribution.

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