ELECTRICITY MARKET DESIGN: MULTI-INTERVAL PRICING MODELS

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Looking Ahead: Price Formation and Multi-Period Dispatch

Harvard Electricity Policy Group

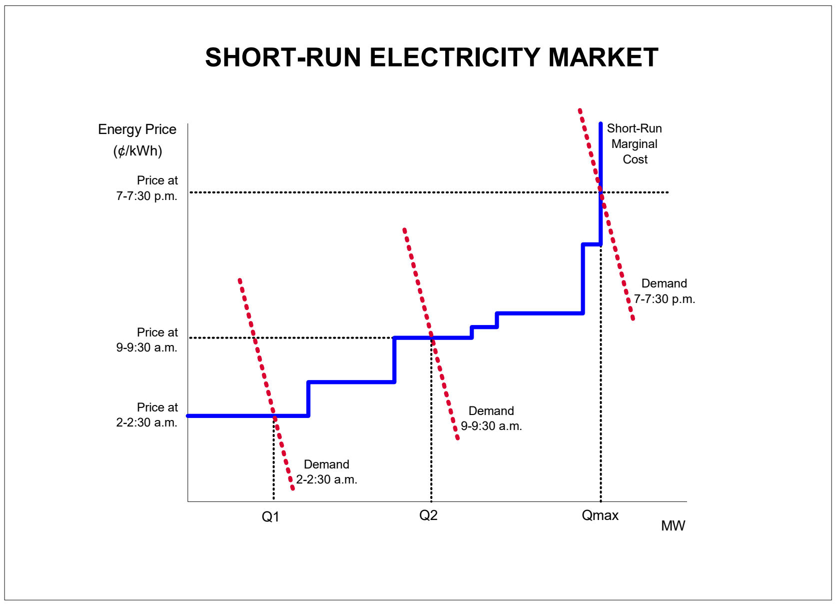
June 22, 2020

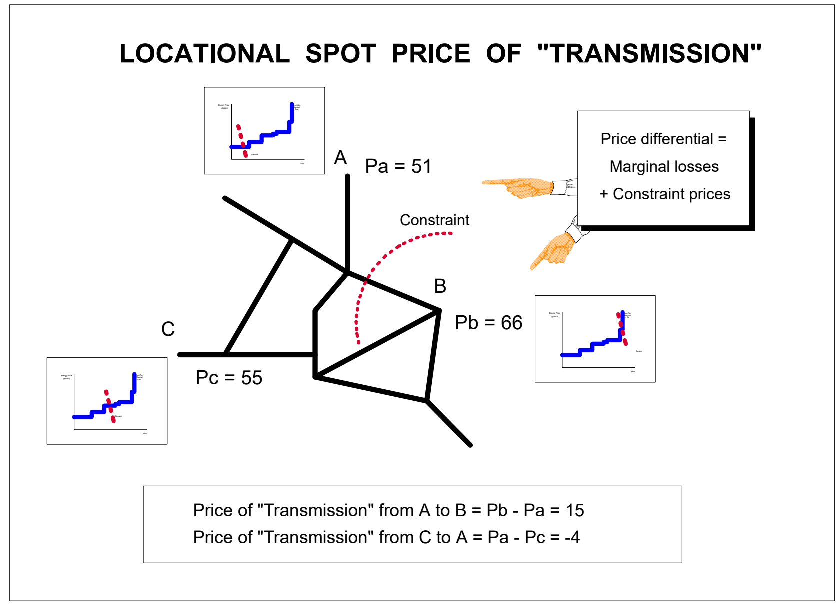
Looking Ahead: Price Formation and Multi-Period Dispatch

The basic model of bid-based, security-constrained, economic dispatch with locational prices is well understood and provides the foundation for efficient pricing. The most common analysis is for a single period with well-behaved bids and offers without uncertainty. With independent dispatches, serial application of this approach produces efficient prices. The real dispatch system requires some degree of look-ahead with intertemporal constraints. The expansion of intermittent resources increases the importance of efficient multi-period pricing. In principle, the same model applies for the multi-period dispatch. Relaxing any of the assumptions, however, presents new challenges for efficient pricing. Rolling dispatches must adjust to uncertain conditions inducing changes over time. Bids and offers with start-up, shut-down, and multiperiod operating constraints require some form of extended locational marginal pricing and associated uplift requirements. Current practices differ across organized market. How important are efficient multi-period prices? What approaches might balance the current competing requirements to deal with efficiency, uncertainty and computational feasibility? What new modeling and software innovations are on the horizon?

The focus here is on the real-time dispatch. There are related issues in day-ahead models.

An efficient short-run electricity market determines a market clearing price based on conditions of supply and demand balanced in an economic dispatch. Everyone pays or is paid the same price. The same principles apply in an electric network. This is the familiar starting point, with many implicit assumptions. (Schweppe, Caramanis, Tabors, & Bohn, 1988)





The general problem of interest is the multi-period commitment and dispatch problem. Assume a DC-Load model with a linear loss approximation. A stylized version of the unit commitment and dispatch problem for a fixed demand $\mathbf{y}$ is formulated in (Gribik, Hogan, & Pope, 2007) **as:**

Constants:	Variables:
$\mathbf{y}_t =$ vector of nodal loads in period t	$start_{it} = \begin{cases} 0 \text{ if unit i is not started in period t} \\ 1 \text{ if unit i is started in period t} \end{cases}$
$m_{it} =$ minimum output from unit i in period t if unit is on	$on_{it} = \begin{cases} 0 \text{ if unit i is off in period t} \\ 1 \text{ if unit i is on in period t} \end{cases}$
$M_{it} =$ maximum output from unit i in period t if unit is on	$g_{it} =$ output of unit i in period t
$ramp_{it} =$ maximum ramp from unit i between period t-1 and period t	$\mathbf{d}_t =$ vector of nodal demands in period t.
$StartCost_{it} =$ Cost to start unit i in period t	
$NoLoad_{it} =$ No load cost for unit i in period t if unit is on	
$\overline{F}_{kt}^{\max} =$ Maximum flow on transmission constraint k in period t.	

$$v\left(\{\mathbf{y}_t\}\right) = \inf_{\text{g,d,on,start}} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)$$

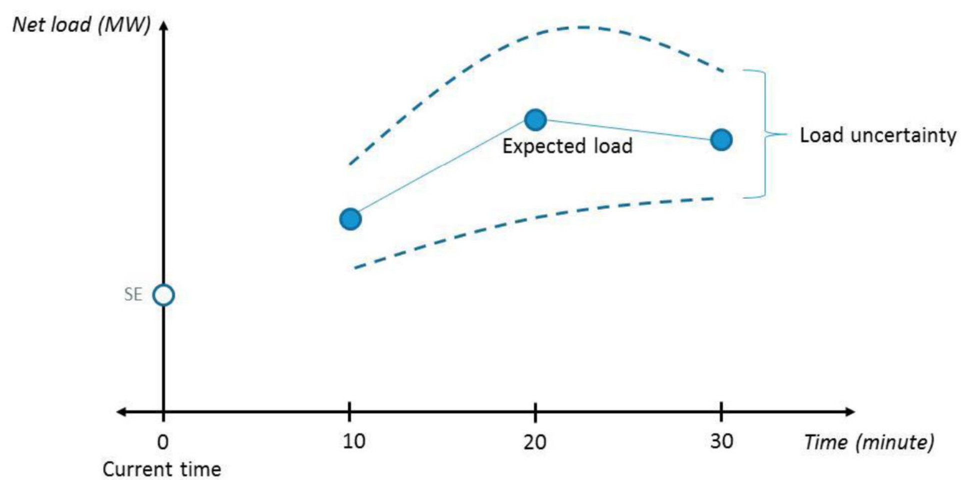
subject to

$$
\begin{array}{ll}
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
\mathbf{d}_t = \mathbf{y}_t & \forall t.
\end{array}
$$

Multi-Period pricing must address both uncertainty and look-ahead dynamics. For a discussion of operating reserves see (Hogan & Pope, 2017)**. The focus here is on deterministic models with rolling updates with the expected load, not on the related questions surrounding stochastic problems and operating reserves.** (Schiro, 2017)

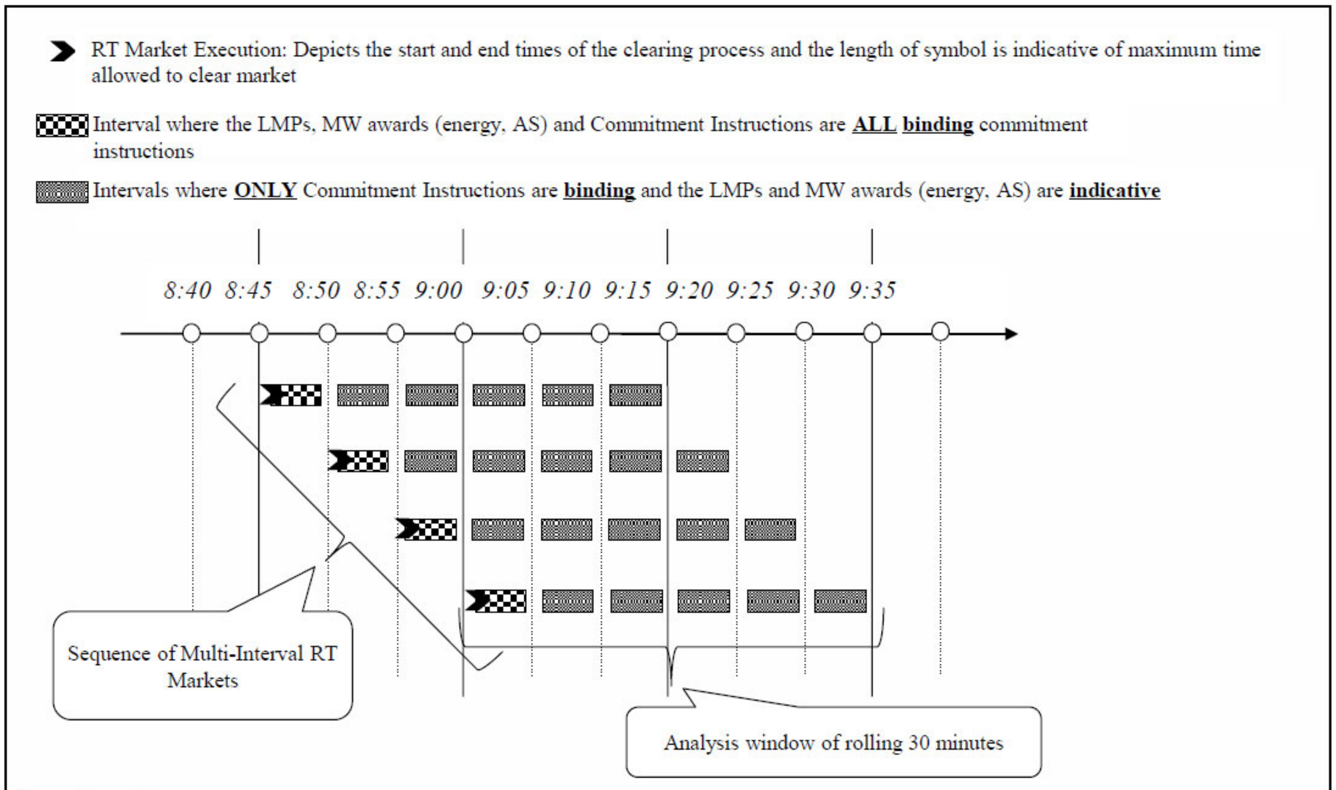
***Multi-period pricing*. Uncertainty**

- Multi-period pricing is useful for expected load changes but may not help with load uncertainty
  - Load uncertainty for Time 10 is handled by AGC
  - Load uncertainty for Times 20-30 can be problematic (economic dispatch runs the system "as lean as possible")



A real-time dispatch model with multiple periods and look ahead is an approach found in some organized markets:

- MIRTM with six 5-minute intervals (total of 30 minutes)



Item 5
ERCOT Public

Joel Mickey, “Multi-Interval Real-Time Market Overview,” Board of Directors Meeting, ERCOT Public, October 13, 2015.

A simple version of a real-time look-ahead dispatch and pricing model with multiple periods might be as in:

$$\begin{aligned}
&\underset{\mathbf{g},\mathbf{d}}{\text{Min}} \sum_{t=1}^{T}\sum_{i} GenCost_{it}\left(g_{it}\right) \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i,t \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
&\mathbf{e}^{T}\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t
\end{aligned}$$

This drops all the commitment decisions, treating them as fixed. For a given end period, this is a basic model. With a subsequent dispatch period, t^{*}, a version of the rolling model might be have a future horizon of fixed length as in the ERCOT example:

$$\begin{aligned}
&\underset{\mathbf{g},\mathbf{d}}{\text{Min}} \sum_{t \ge t^{*}}^{T+t^{*}}\sum_{i} GenCost_{it}\left(g_{it}\right) \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i,t \ge t^{*} \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \ge t^{*} \\
&\mathbf{e}^{T}\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^{*} \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t \ge t^{*} \\
&g_{i,t^{*}-1} = g_{i,t^{*}-1}^{*} && \forall i
\end{aligned}$$

The full rolling model creates end-point problems. To further simplify the present discussion, consider the special case where the end-point is fixed at T, perhaps far ahead. This produces the "truncated rolling model" for the remaining periods of the dispatch.

$$
\begin{array}{ll}
\underset{g,d}{\text{Min}} \sum_{t \geq t^*}^{T} \sum_i GenCost_{it}\left(g_{it}\right) & \\
\text{subject to} & \\
m_{it} \leq g_{it} \leq M_{it} & \forall i, t \geq t^* \\
-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} & \forall i, t \geq t^* \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \geq t^* \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \leq \overline{F}_{kt}^{\max} & \forall k, t \geq t^* \\
g_{i,t^*-1} = g^*_{i,t^*-1} & \forall i
\end{array}
$$

The rolling dispatch model incorporates a revised but still deterministic forecast for dispatch conditions. The rolling pricing model produces updated prices. A proposed minimal condition is to require a pricing method that preserves price consistency.

> **Price Consistency:** With no change in the forecast conditions, the truncated rolling pricing model produces no change in the prices.

This is a natural but strong condition that defines the acceptable pricing model.

The analysis of basic and truncated rolling models is affected by different cases. The discussion here focuses on three different dispatch types that affect the interpretation and the pricing.

- **Single Period.** The standard model for a one dispatch period. This can include commitment decisions with start up costs, minimum loads, and other complexities. This model stands behind much of the analysis and intuition.

- **Independent Multi-Period.** The multi-period dispatch reduces to a collection of independent dispatch models with no intertemporal interaction. This excludes ramping constraints, startup decisions that last more than a single period, and so on. This model is useful for isolating the important conditions that affect the full dependent multi-period case.

- **Dependent Multi-Period.** The standard model but with intertemporal interactions from startup and shutdown decisions, minimum run times, ramping constraints, and so on.

ELECTRICITY MARKET — Real-Time Pricing

The interest is in feasible pricing models that support the economic dispatch, have limited uplift, and meet the price consistency test.

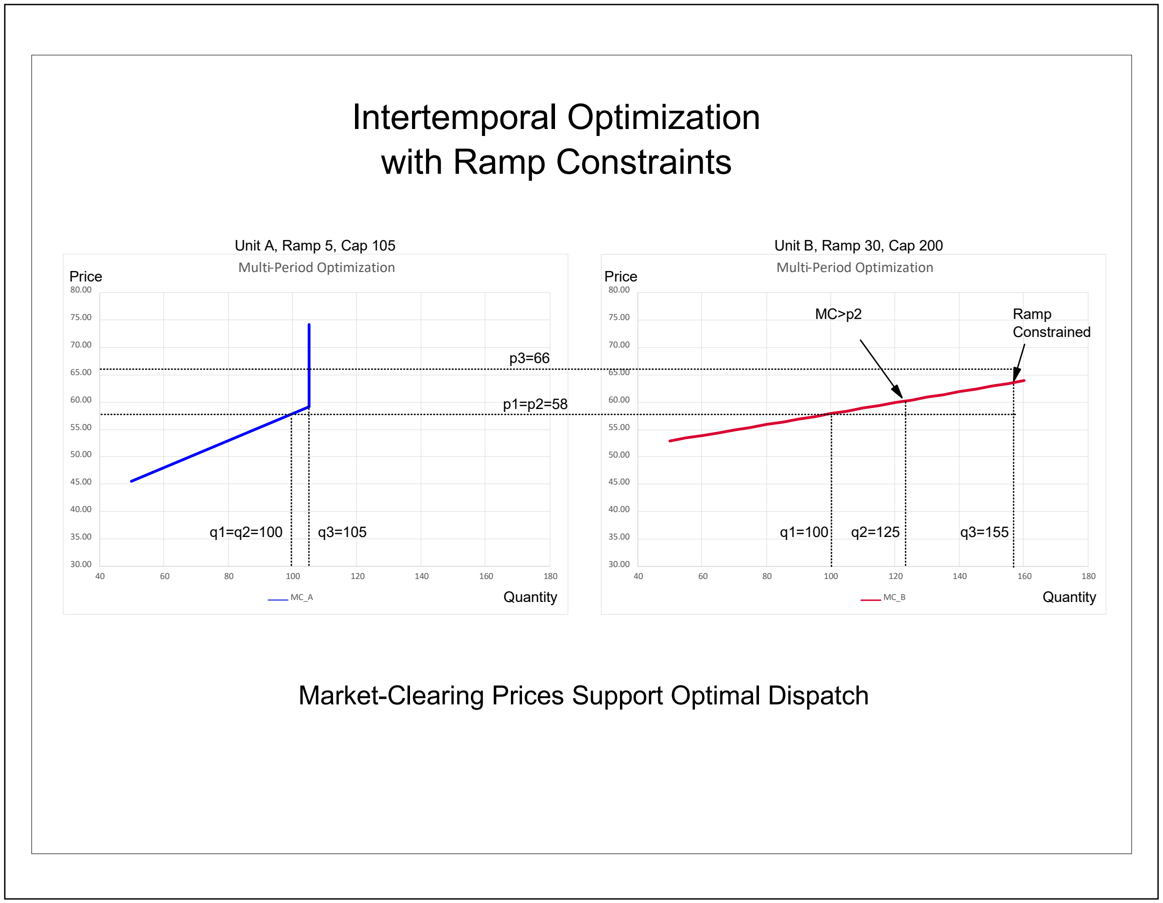
	Basic			Truncated Rolling	
	Single Period	**Independent Multi-Period**	**Dependent Multi-Period**	**Independent Multi-Period**	**Dependent Multi-Period**
Strictly Convex (e.g. linear supply curves)					
General Convex (e.g. step function supply)					
Other (e.g. start-up cost, minimum generation, etc.)					

In the fully convex cases, the pricing models almost reduce to the standard interpretation of LMP pricing, with the prices equal to the marginal cost from the (intertemporal) optimization.

	Basic			Truncated Rolling	
	Single Period	**Independent Multi-Period**	**Dependent Multi-Period**	**Independent Multi-Period**	**Dependent Multi-Period**
Strictly Convex (e.g. linear supply curves)	**LMP**	**LMP**	**LMP**	**LMP**	**LMP** (Biggar & Hesamzadeh, 2020)
General Convex (e.g. step function supply)	**LMP** (Schweppe et al., 1988)	**LMP**	**LMP**	**LMP**	**Constrained LMP** (Hua et al., 2019)
Other (e.g. start-up cost, minimum generation, etc.)					

In the fully convex cases, the basic dependent multi-period prices follow the LMP formulation, but prices do not always equal generator bid-in marginal costs.

An intertemporal optimization with ramp constraints. Consider two generating units with the indicated marginal cost curves. The three-period problem aggregate demands are 200, 225, and 260 MW respectively. The market clearing price is $58/MWh in the first two periods, and increases to $66/MWh in the third period.



Unit A always produces at the market price (in the first two periods) or at capacity. This is the normal condition when the ramping constraint is not binding and price equals bid-in marginal cost. However, Unit B increases production to 125 MW in the second period even though its bid-in marginal cost of $59.25/MWh at that level is above the market clearing price. This puts Unit B in a position to ramp up to 155 MW in the third period, when the price is $66/MWh and its direct marginal cost is $63.50/MWh. The difference of $2.50 is the shadow price of the binding ramp constraint for Unit B in the third period. This second period production for Unit B would be "out-of-market" if there were no intertemporal interactions. But for the intertemporal optimization, this production profile is consistent with the market outcome. Both for Unit A and Unit B the optimal dispatch is also the profit maximizing choice given the market-clearing price. As a price-taker, Unit B would recognize its ramp constraints and choose to follow the economic dispatch.

For the general problem, convex hull (CH) pricing minimizes uplift. The integer relaxation (IR) or dispatchable model allows for continuous commitment variables. The IR is a standard convex optimization problem. Assume the formulation has a convex and homogeneous objective function. (MISO, 2019)

$$
\begin{aligned}
&\underset{\text{g,d,on,start}}{Min} \left(\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)\right) \\
&\text{subject to} \\
&m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
&start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
&0 \le start_{it} \le 1 && \forall i,t \\
&0 \le on_{it} \le 1 && \forall i,t \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t
\end{aligned}
$$

The ramping constraints are an important complication.

- **With no intertemporal constraints.** The LMP prices from the relaxed problem are the same as the convex hull prices. (Chao, 2019)
- **With ramping constraints.** The IR LMP prices are good approximate convex hull prices. There are additional valid inequalities that can be included to strengthen the approximation. (Hua et al., 2019)

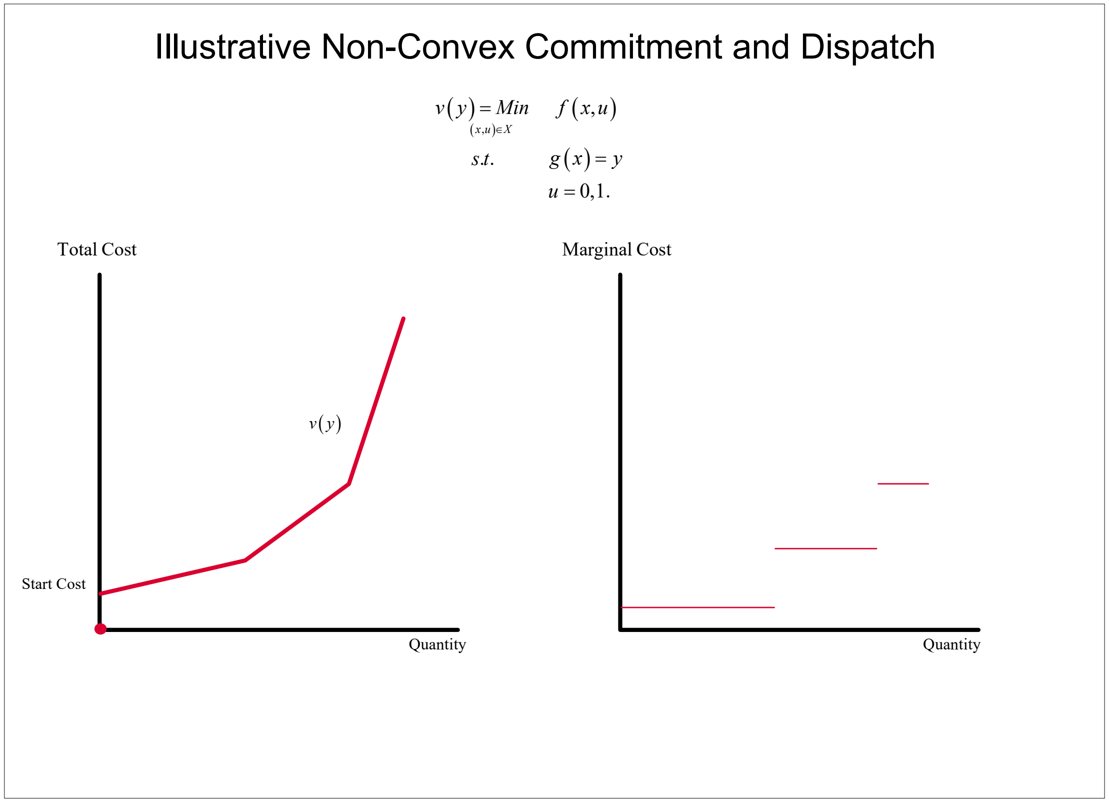
The IR problem is easy to solve, uses the exact formulation of the corresponding dispatch model, but treats the commitment variables as continuous.

The interest is in feasible pricing models that support the economic dispatch, have limited uplift, and meet the price consistency test.

	Basic			**Truncated Rolling**	
	Single Period	**Independent Multi-Period**	**Dependent Multi-Period**	**Independent Multi-Period**	**Dependent Multi-Period**
Strictly Convex (e.g. linear supply curves)	**LMP**	**LMP**	**LMP**	**LMP**	**LMP** (Biggar & Hesamzadeh, 2020)
General Convex (e.g. step function supply)	**LMP** (Schweppe et al., 1988)	**LMP**	**LMP**	**LMP**	**Constrained LMP** (Hua et al., 2019)
Other (e.g. start-up cost, minimum generation, etc.)	**IR** Model may equal CH. Uplift. (Chao, 2019)	**IR** Model may equal CH. Uplift.	**IR** Model dependent. Uplift.	**IR** Model may equal CH. Uplift.	**Constrained IR** Model dependent. Uplift.
	CH & Min Uplift (Gribik et al., 2007)	**CH & Min Uplift**	**CH & Min Uplift**	**CH & Min Uplift**	**Look-Back CH & Min Uplift** (Hogan, 2016)

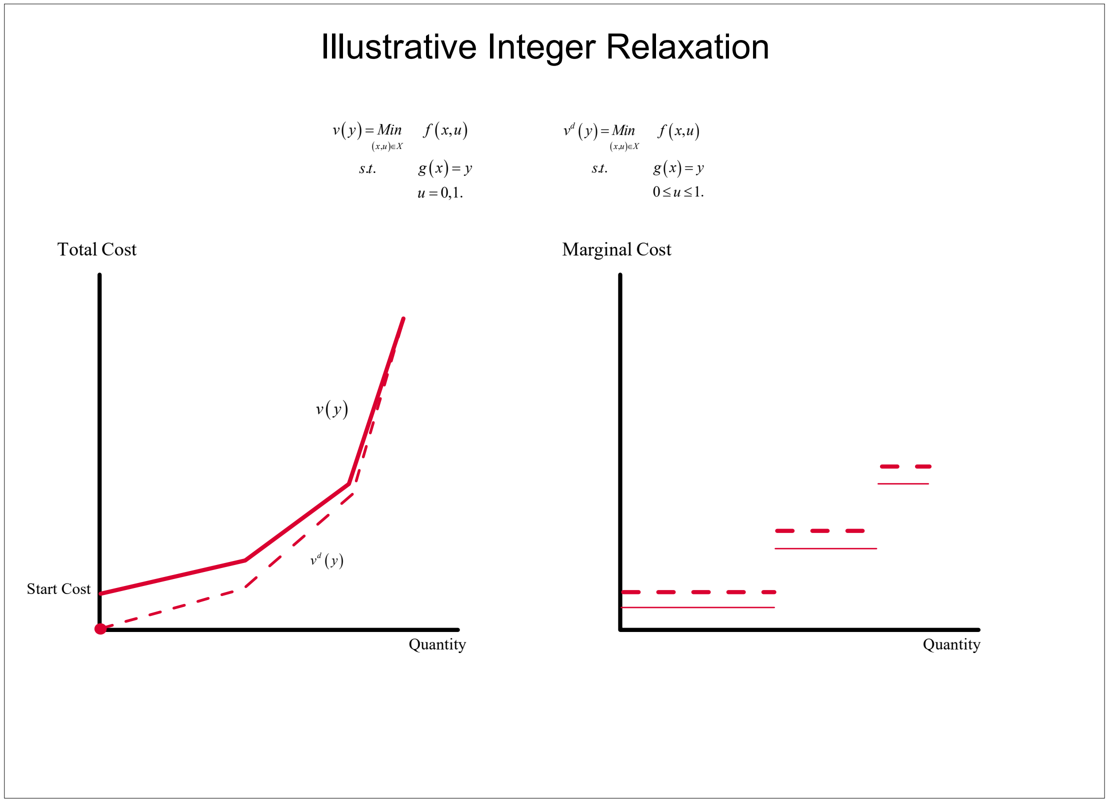
A simple example illustrates connections among the different pricing model formulations that affect the integer relaxation approach.

$$
\begin{aligned}
v(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\
& \sum_{i=1}^{n} x_i = y \\
s.t. \quad & 0 \le x_i \le X_i \\
& \sum_{i=1}^{n} x_i \le uK \\
& u = 0,1.
\end{aligned}
$$

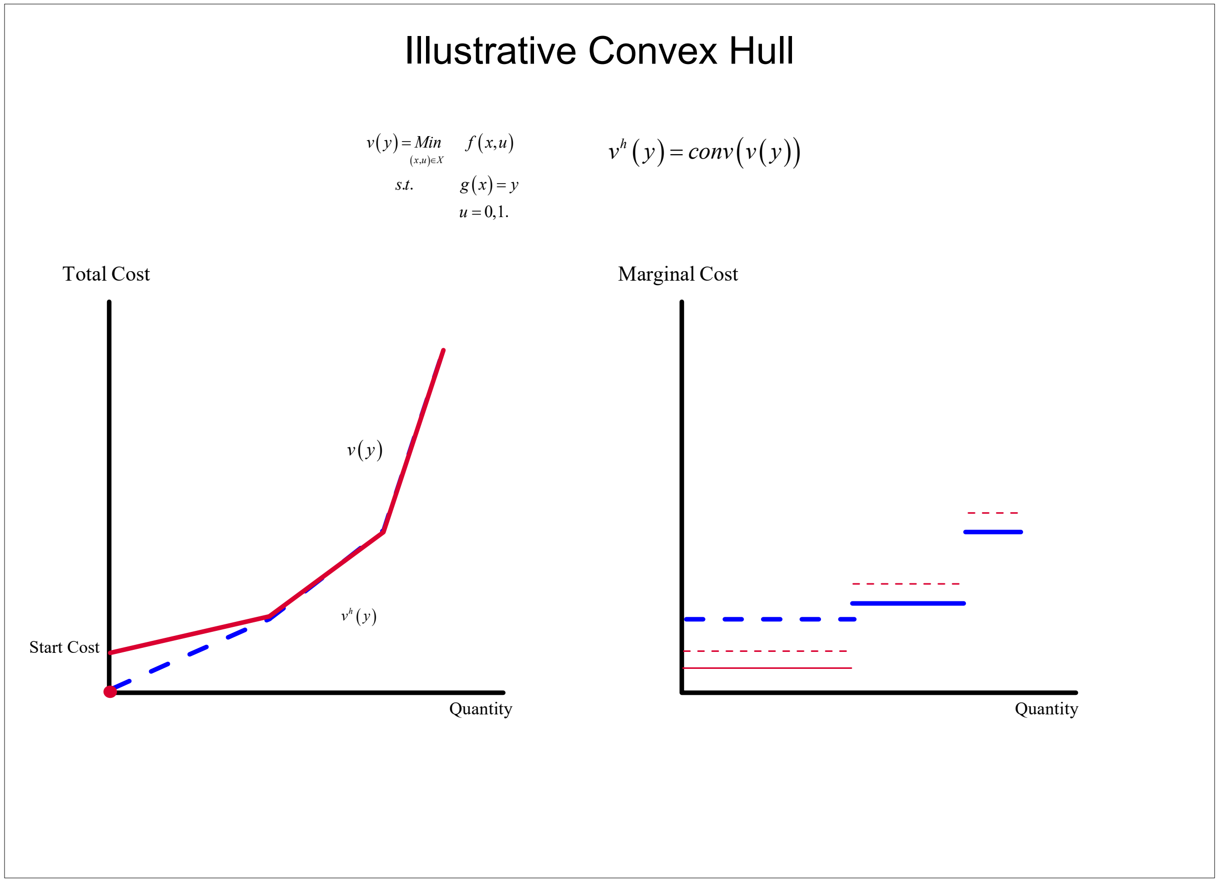


A easy dispatchable model applies an integer relaxation.

$$
\begin{aligned}
v^d(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\
& \sum_{i=1}^{n} x_i = y \\
s.t. \quad & 0 \le x_i \le X_i \\
& \sum_{i=1}^{n} x_i \le uK \\
& 0 \le u \le 1.
\end{aligned}
$$



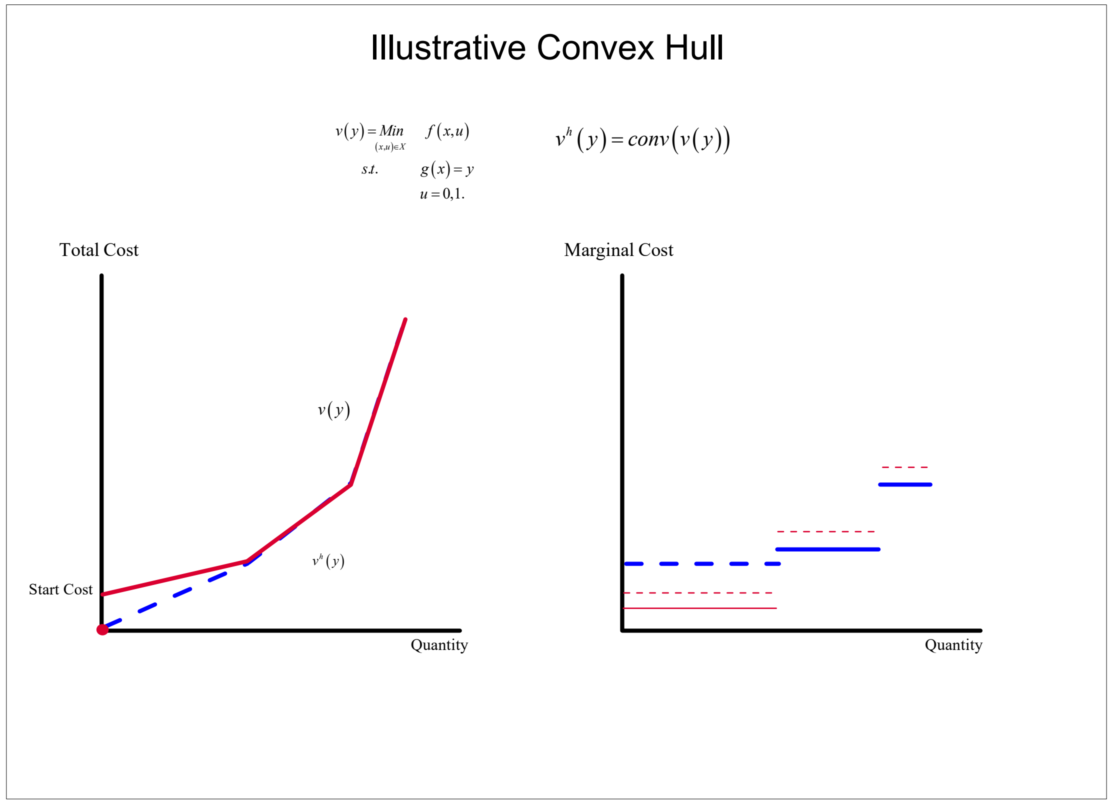
The convex hull or minimum uplift model provides the best convex approximation to the total cost function.



An alternative description of the model with the same solutions provides a different integer relaxation. (Chao, 2019) **In this case the integer relaxation produces the convex hull. In general, there are many ways to change the formulation of the original model without affecting the dispatch solutions but producing different price approximation.** (Zheng, Zhao, Schiro, & Litvinov, 2018)

$$
\begin{aligned}
v(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\
s.t. \quad & \sum_{i=1}^{n} x_i = y \\
& 0 \le x_i \le u X_i \\
& u = 0,1.
\end{aligned}
$$

$$
\begin{aligned}
v^d(y) = \underset{(x,u)}{Min} \quad & \sum_{i=1}^{n} c_i x_i + ku \\
s.t. \quad & \sum_{i=1}^{n} x_i = y \\
& 0 \le x_i \le u X_i \\
& 0 \le u \le 1.
\end{aligned}
$$



A real-time pricing model involves multiple periods and look ahead. Applying an Extended LMP framework involves choices about what is fixed and what is variable. Natural principles suggested include:

- **Real-time quantity anchor.** Conditioning to reflect evolving economic dispatch and commitment. For example, the pricing dispatch would account for ramping limits that constrain the degree that the pricing dispatch could deviate from the actual dispatch to ensure that the price market-clearing dispatch would always be feasible conditioned on the actual dispatch.
- **Real-time price consistency.** Given perfect foresight, where actual conditions equal the forecast conditions, the methodology produces the same set of prices.

For actual commitment and dispatch, past decisions are sunk and real-time quantity anchors apply.

The pricing model could employ more flexibility. The ELMP approach in general incorporates intertemporal constraints, past decisions are important, and reflects fixed costs of units not committed.

Apparently, without full convexity, there must be a choice of which principle to apply in the pricing model. Under both CH and IR, the pricing model dispatch deviates from the physical dispatch.

The minimum uplift or convex hull prices present a computational challenge.

- **Calculation of the Lagrangean Dual Solution.** Works in theory but is slow to converge and inevitably leads to numerical approximation.
- **New Methods.** Focus on primal construction of the convex hull.
    - **Primal Convex Hull.** Constructing the convex hulls of the components and then optimizing the resulting problem. Provides exact solutions for a useful class of problems, and reports good approximations in other case. (Hua & Baldick, 2017)
    - **Expanded Unit Commitment.** Adding constraints and variables, constructing a master problem that characterizes the convex hull via Benders Decomposition. Provides exact solution and reports good computational performance. (Knueven, Ostrowski, & Wang, 2018) (Knueven, Ostrowski, Castillo, & Watson, 2019) (Bacci, Frangioni, Gentile, & Tavlaridis-Gyparakis, 2019)
- **Integer Relaxation.** How best to formulate equivalent models? (Chao, 2019) How close is close enough in approximating CH prices?

Alternative pricing models have different features and raise additional questions for both dispatch and pricing.

- **The Full Rolling Model.** The truncation assumption simplifies the analysis and provides an approximation for the true rolling model. Extending the endpoint alters the dispatch and prices.
- **Uncertainty.** As the dispatch rolls forward, new information arises and the forecast changes. The pricing model can create new prices. This leads to uplift requirements to ensure that the prices support and associated uplift support the actual dispatch.
- **Rolling Dispatch with Endpoint Constraints.** A rolling real-time dispatch model is constrained to match the real-time past and DA future after the LA horizon, which should be the best available estimate of the future conditions. "If the realization deviates significantly from the [DA] forecast value, then the estimations become inaccurate, [DA] re-optimization should be performed." (Zhao, Zheng, & Litvinov, 2019)
- **Operating Reserve Demand.** All models compatible with existing and proposed operating reserve demand curves.
- **Financial Transmission Rights.** Transmission revenue collected under the market clearing solution would be sufficient to meet the obligations under the FTRs. However, this may not be true for the revenues under the economic dispatch, which is not a market clearing solution at the ELMP prices, even though the FTRs are simultaneously feasible. The FTR uplift amount included in the decomposition of the total uplift that is minimized by the CH prices. This uplift payment would be enough to ensure revenue adequacy of FTRs under CH pricing. (Cadwalader, Gribik, Hogan, & Pope, 2010)
- **Day-ahead and real-time interaction.** With uncertainty in real-time and virtual bids, expected real-time price is important, and may be similar under all pricing models.

APPENDIX

Rolling forward to the t* interval, with prior dispatch $g_{i,1}^{*} \cdots g_{i,t^*-1}^{*}$. A Look Ahead (LA) dispatch model has:

$$
\begin{aligned}
&\underset{\mathrm{g,d}}{\mathrm{Min}} \; \sum_{t \ge t^*} \sum_{i} GenCost_{it}\left(g_{it}\right) \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t \ge t^* \\
&-ramp_{it} \le g_{it^*} - g_{i,t^*-1}^{*} \le ramp_{it} && \forall i, t^* \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t > t^* \\
&\mathbf{e}^{T}\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^* \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^*
\end{aligned}
$$

This model will produce LMP values for the future periods. However, even in the fully convex case with perfect foresight, this model may produce prices that are not consistent over time and do not support the dispatch. (Hua et al., 2019)

Part of the difficulty arises from the generality of convex generation offer functions that can create dual degeneracy in the supply curves. With strictly convex cost functions, implying continuous offer curves rather than step functions, the "time inconsistency problem disappears." (Biggar & Hesamzadeh, 2020)

This clarifies the source of the difficulty. However, given the ubiquitous use of step-function generator offer curves, the price consistency problem remains.

A modified version of a rolling pricing model addresses this price consistency problem in the convex case. The essence of a more general proposal of (Hua et al., 2019) **is to use the dual or shadow prices from the prior LA dispatch model to modify the objective function for the current LA pricing model.**

Accompanying the LA dispatch model, with up and down ramp shadow prices $\mu_{t^*-1}^{ur*}, \mu_{t^*-1}^{dr*}$ **from the prior pricing run, set the (separate but related) LA pricing as:**

$$\begin{aligned}
&\underset{\mathrm{g,d}}{\text{Min}} \sum_{t \ge t^*} \sum_i GenCost_{it}\left(g_{it}\right) + \sum_i \mu_{i,t-1}^{ur*} g_{it^*} - \sum_i \mu_{i,t-1}^{dr*} g_{it^*} \\
&\text{subject to} \\
&m_{it} \le g_{it} \le M_{it} && \forall i, t \ge t^* \\
&-ramp_{it} \le g_{it^*} - g_{i,t^*-1}^{*} \le ramp_{it} && \forall i, t^* \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i, t > t^* \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \ge t^* \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k, t \ge t^*
\end{aligned}$$

The shadow price from the prior LA pricing model reflects the opportunity cost over past decisions. This "prices out the past" and preserves price consistency in the perfect foresight case.

The ZZL model modifies both the actual dispatch formulation and the associated pricing model. The dispatch model uses the solution from the (longer horizon) day-ahead market to constrain the starting and terminal conditions for the real-time LA dispatch.

Suppose the LA dispatch terminate at t. Then let g_{i,t^*+1}^{DA} be the solution from the next period for the day-ahead market. The essence of the rolling ZZL dispatch is**

$$
\begin{array}{ll}
\underset{\mathrm{g,d}}{\mathrm{Min}} \sum_{t \geq t^*} \sum_{i} GenCost_{it}\left(g_{it}\right) & \\
\text{subject to} & \\
m_{it} \leq g_{it} \leq M_{it} & \forall i, t^* \leq t \leq t^{**} \\
-ramp_{it} \leq g_{it^*} - g_{i,t^*-1}^{*} \leq ramp_{it} & \forall i, t^* \\
-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} & \forall i, t^* \leq t \leq t^{**} \\
-ramp_{it} \leq g_{t^{**}+1}^{DA} - g_{i,t-1} \leq ramp_{it} & \forall i, t^{**}+1 \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \geq t^* \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \leq \overline{F}_{kt}^{\max} & \forall k, t \geq t^*
\end{array}
$$

Hence, the rolling real-time dispatch model is constrained to match the real-time past and DA future after the LA horizon, which should be the best available estimate of the future conditions.

“If the realization deviates significantly from the [DA] forecast value, then the estimations become inaccurate, [DA] re-optimization should be performed.” (Zhao et al., 2019)

The ZZL pricing model proposal modifies both the actual dispatch formulation and the associated pricing model. The pricing dispatch model uses the dual or shadow prices from the (longer horizon) day-ahead market price out the past and the future.

With ramp shadow prices from the day-ahead solution $\mu_{t^*-1}^{urDA}, \mu_{t^*-1}^{drDA}, \mu_{t^{**}+1}^{urDA}, \mu_{t^{**}+1}^{drDA}$, set the (separate but related) LA pricing as:[1]

$$
\begin{array}{ll}
\underset{\mathrm{g,d}}{\mathrm{Min}} \sum_{t \geq t^*} \sum_{i} GenCost_{it}\left(g_{it}\right) + \sum_{i} \mu_{i,t-1}^{urDA} g_{it^*} - \sum_{i} \mu_{i,t-1}^{drDA} g_{it^*} - \sum_{i} \mu_{i,t^{**}+1}^{urDA} g_{it^{**}} + \sum_{i} \mu_{i,t^{**}+1}^{drDA} g_{it^{**}} & \\
\text{subject to} & \\
m_{it} \leq g_{it} \leq M_{it} & \forall i, t^* \leq t \leq t^{**} \\
-ramp_{it} \leq g_{it} - g_{i,t-1} \leq ramp_{it} & \forall i, t^* \leq t \leq t^{**} \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \geq t^* \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \leq \overline{F}_{kt}^{\max} & \forall k, t \geq t^*
\end{array}
$$

Hence, this preserves the computational advantage of a shorter dispatch period by using the day-ahead solution to connect the present to the past and future. With perfect foresight, this achieves price consistency.

[1] This differs from the LA model of (Hua et al., 2019) in the treatment of the priced-out constraints. For the perfect foresight analysis, these constraints are redundant.

The ELMP model applied to the stylized unit commitment problem employs the dual prices from a particular Lagrangean relaxation.

$$
v^h\left(\{\mathbf{y}_t\}\right) \equiv
\sup_p \left\{
\begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{
\begin{array}{ll}
\inf\limits_{\text{g,d,on,start}} \left(
\begin{array}{l}
\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\
-\sum_t \mathbf{p}_t^T \mathbf{d}_t
\end{array}
\right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t
\end{array}
\right\}
\end{array}
\right\}
$$

The ELMP price is determined for all periods as the pricing solution to this problem.

The ELMP is a solution $\mathbf{p}$ for the dual or convex hull problem with the loss and transmission limits included as constraints. A "market-clearing" solution is a solution to the inner problem for given prices $\mathbf{p}$.

$$
v^h\left(\{\mathbf{y}_t\}\right) \equiv
$$

$$
\sup_p \left\{ \begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{ \begin{array}{ll}
\inf\limits_{\text{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t
\end{array} \right\}
\end{array} \right\}
$$

A proposal for real-time price consistency in ELMP is to fix past decisions in the inner "market clearing" solution, as well as fixing the prices. Hence, the conditional market-clearing pricing model at time τ would take the determined prices $p_1^*, p_2^*, \cdots p_{\tau-1}^*$ and market clearing dispatch $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ for the prior periods as fixed and solve as the pricing model:

$$
v^\tau\left(\{\mathbf{y}_t\}\right) \equiv
$$

$$
\sup_p \left\{
\begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{
\begin{array}{ll}
\inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
z^{\tau-1} = z^{*\tau-1} &
\end{array}
\right\} \\
p_t = p_t^*, t \le \tau - 1
\end{array}
\right\}
$$

However, the hoped for price consistency depends on separability across periods. The general problem is not separable, and fixing $z^\tau = \{\mathrm{g}_t, \mathrm{d}_t, \mathrm{on}_t, \mathrm{start}_t\}_1^\tau$ does not ensure price consistency.

A sufficient condition for real-time price consistency in ELMP is that all commitment and dispatch variables that are in the economic dispatch or are assigned an uplift payment from the market-clearing solution be included in the pricing model. This allows for slowly pruning those offers that were not committed in either the economic commitment or the market-clearing commitment and are subsequently excluded from retroactive starts ($Excluded_\tau$). Hence, the conditional dual pricing model at time τ could take as determined prices the prior periods $p_1^*, p_2^*, \cdots p_{\tau-1}^*$:

$$
v^\tau\left(\{\mathbf{y}_t\}\right) \equiv \sup_p \left\{ \begin{array}{l}
+\sum_t \mathbf{p}_t^T \mathbf{y}_t \\
+\left\{ \begin{array}{ll}
\inf\limits_{\mathrm{g,d,on,start}} \left(\begin{array}{l} \sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum_t \mathbf{p}_t^T \mathbf{d}_t \end{array} \right) & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
start_{it} = 0 \text{ or } 1 & \forall i,t \\
on_{it} = 0 \text{ or } 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
start_{it} = 0, i \in Excluded_\tau &
\end{array} \right\} \\
p_t = p_t^*, t \le \tau - 1
\end{array} \right\}
$$

Different formulations of the unit commitment problem yield the same convex hull but have different integer relaxations.

Suppose that $GenCost_{it}(g_{it})$ **is convex and homogeneous of degree one.**[2] **This is a very weak condition. Suppose we have a commitment variable and a piecewise representation of generation cost over intervals** $\left[0, X_j\right], \sum_j X_j = M$ **.**

A: $\widetilde{GenCost}(g,z) = \underset{x}{Min}\left\{\sum_j c_j x_j \middle| 0 \le x_j \le X_j, \sum_j x_j = g, 0 \le g \le zM\right\}$ **is not homogeneous.**

B: $GenCost(g,z) = \underset{x}{Min}\left\{\sum_j c_j x_j \middle| 0 \le x_j \le zX_j, \sum_j x_j = g, 0 \le g \le M\right\}$ **is homogeneous.**

These functions agree when $z = 0,1$**, so they have the same convex hull.**

- Without ramping constraints, unimodular constraints on commitment variables assure that integer relaxation with model **B** provides convex hull prices. (Chao, 2019)
- With ramping constraints, an expanded unit commitment characterization can employ a variant of model **B** and provide convex hull prices. (Yu, Guan, & Chen, 2019)

---

2 Homogeneous of degree one: $GenCost_{it}(\alpha g_{it}) = \alpha GenCost_{it}(g_{it}), \alpha \ge 0.$

The integer relaxation or dispatchable model allows for continuous commitment variables. This is a standard convex optimization problem. Assume the formulation has a convex and homogeneous objective function. (MISO, 2019)

$$
\begin{aligned}
&\underset{\mathrm{g,d,on,start}}{Min} \left(\sum_t \sum_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right)\right) \\
&\text{subject to} \\
&m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} && \forall i,t \\
&-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} && \forall i,t \\
&start_{it} \le on_{it} \le start_{it} + on_{i,t-1} && \forall i,t \\
&0 \le start_{it} \le 1 && \forall i,t \\
&0 \le on_{it} \le 1 && \forall i,t \\
&\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 && \forall t \\
&Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} && \forall k,t
\end{aligned}
$$

The ramping constraints are an important complication.

- **With no ramping constraints.** The LMP prices from the relaxed problem are the same as the convex hull ELMP prices. (Chao, 2019)
- **With ramping constraints.** The LMP prices are good approximate convex hull prices. There are additional valid inequalities that can be included to strengthen the approximation. (Hua et al., 2019)

The relaxed problem is easy to solve, uses the exact formulation of the corresponding dispatch model but treats the commitment variables as continuous.

An adaptation of the sequential model for the rolling LA pricing with the relaxed approximation of the pricing problem presents a relative simply tool. First we fix the prices for prior periods and price out the constraints to include them as part of the objective function. Then we utilize the relaxed model to find the approximate prices:

$$
\begin{array}{ll}
v^{r\tau}\left(\{\mathbf{y}_t\}\right) \equiv & \\
\inf\limits_{\mathrm{g,d,on,start}} \begin{pmatrix} \sum\limits_t \sum\limits_i \left(StartCost_{it} \cdot start_{it} + NoLoad_{it} \cdot on_{it} + GenCost_{it}\left(g_{it}\right)\right) \\ -\sum\limits_{t \le \tau - 1} \mathbf{p}_t^{*T}\left(\mathbf{d}_t - \mathbf{y}_t\right) \end{pmatrix} & \\
\text{subject to} & \\
m_{it} \cdot on_{it} \le g_{it} \le M_{it} \cdot on_{it} & \forall i,t \\
-ramp_{it} \le g_{it} - g_{i,t-1} \le ramp_{it} & \forall i,t \\
start_{it} \le on_{it} \le start_{it} + on_{i,t-1} & \forall i,t \\
0 \le start_{it} \le 1 & \forall i,t \\
0 \le on_{it} \le 1 & \forall i,t \\
\mathbf{e}^T\left(\mathbf{g}_t - \mathbf{d}_t\right) - LossFn_t\left(\mathbf{d}_t - \mathbf{g}_t\right) = 0 & \forall t \\
Flow_{kt}\left(\mathbf{g}_t - \mathbf{d}_t\right) \le \overline{F}_{kt}^{\max} & \forall k,t \\
\mathbf{d}_t = \mathbf{y}_t & \forall t \ge \tau.
\end{array}
$$

This is a much easier problem to solve than the ELMP. Absent ramping constraints, it yields the convex hull prices. With ramping constraints, it should provide a good approximation.

Since the integer relaxation is a convex problem, the various LA pricing and settlement approximations proposed for LMP in the convex real-time model without commitment decisions could be implemented with an appropriate integer relaxation pricing model to account for commitment decisions.

The discussion of pricing enhancements and these alternative models identifies common objections.

- **Not Using Real Marginal Costs.** The shorthand that short-run efficient prices equal marginal costs depends on an underlying assumption of convexity. In the non-convex case, there may be no efficient linear prices that clear the market.

- **Dispatch and Pricing are Inconsistent.** Only under the assumption of convexity will the marginal cost prices from the dispatch also support the dispatch solution and clear the market. The market design seeks an efficient commitment and dispatch solution. The pricing and uplift payment model is related but not identical under conditions where there is a duality gap.

- **The Implied Dispatch in the Pricing Model Creates Congestion.** Absent convexity, and with a duality gap, the dispatch in the pricing model can violate existing or encounter new constraints. This is a feature, but not a bug. The prices are relevant, not the dispatch.

- **The Prices and Uplift Will Create Perverse Incentives.** The purpose of the uplift payments is precisely to remove the most perverse incentives not to follow the commitment and dispatch. The mechanism for pricing and uplift is “almost” incentive compatible in a manner like other “first -price” auction frameworks.

- **Self-Scheduled Units Will Manipulate Prices.** The pricing model only employs bids and offers included in the commitment and dispatch model. Competitive self-scheduled units have an incentive to participate in the dispatch.

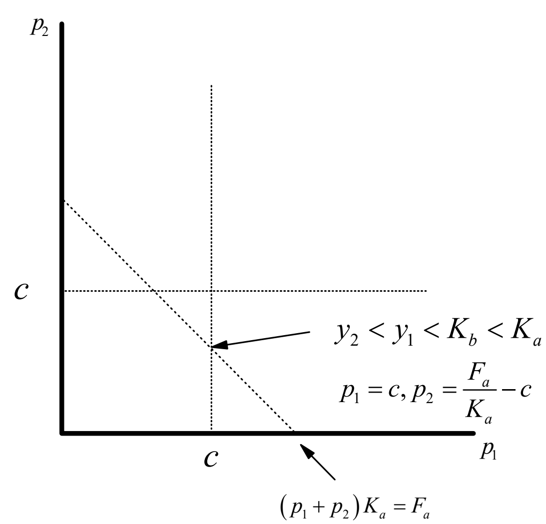
- **Prices Will Go Up.** This is an empirical question. If the efficient energy prices under ELMP are higher, that is a solution and not a problem.

A two period example illustrates the solution and properties of pricing model. The simplified structure with only fixed costs for one plant and variable costs for the other allows us to determine the solution from the graph of critical regions in the dual space of the prices.

Two-Period Price Illustration

Plant a has only fixed costs,
b has only variable costs.

$$
\begin{aligned}
v(y) = &\underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) \\
&s.t. \\
&0 \le x_{a1} \le K_a u_{a1} \\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2}) \\
&0 \le x_{b1} \le K_b \\
&0 \le x_{b2} \le K_b \\
&u_{a1} = 0,1 \\
&u_{a2} = 0,1 \\
&x_{a1} + x_{b1} = y_1 \\
&x_{a2} + x_{b2} = y_2.
\end{aligned}
$$



Dual Problem

$$
v^h(y) = \underset{p_1,p_2}{Max} \left(\begin{array}{l} p_1 y_1 + p_2 y_2 - \\ \left[\begin{array}{l} Max\left((p_1 - c)K_b, 0\right) + \\ Max\left((p_2 - c)K_b, 0\right) + \\ Max\left((p_1 + p_2)K_a - \mathrm{F}_a, 0\right) \end{array} \right] \end{array} \right)
$$

The dual of this problem reduces to a simple solution.

Two-Period Price Illustration

$$
\begin{aligned}
v(y) = &\underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2}}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) \\
&s.t. \\
&0 \le x_{a1} \le K_a u_{a1} \\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2}) \\
&0 \le x_{b1} \le K_b \\
&0 \le x_{b2} \le K_b \\
&u_{a1} = 0,1 \\
&u_{a2} = 0,1 \\
&x_{a1} + x_{b1} = y_1 \\
&x_{a2} + x_{b2} = y_2.
\end{aligned}
$$

Dual Problem

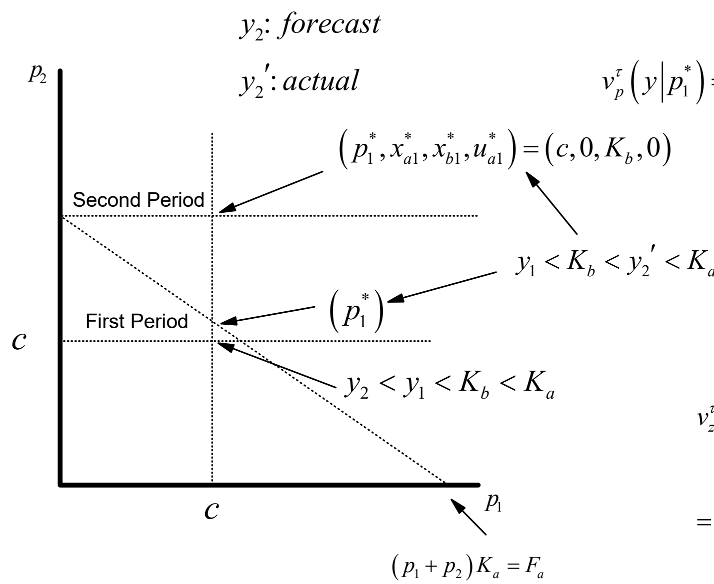
$$
v^h(y) = \sup_p \left\{
\begin{aligned}
&py + \underset{x_{a1},x_{a2},u_{a1},u_{a2},x_{b1},x_{b2},d_1,d_2}{Min} \quad F_a \max(u_{a1},u_{a2}) + c(x_{b1}+x_{b2}) - pd \\
&s.t. \\
&0 \le x_{a1} \le K_a u_{a1} \\
&0 \le x_{a2} \le K_a \max(u_{a1},u_{a2}) \\
&0 \le x_{b1} \le K_b \\
&0 \le x_{b2} \le K_b \\
&u_{a1} = 0,1 \\
&u_{a2} = 0,1 \\
&x_{a1} + x_{b1} = d_1 \\
&x_{a2} + x_{b2} = d_2.
\end{aligned}
\right\}
$$

$$
v^h(y) = \underset{p_1,p_2}{Max} \left(
\begin{aligned}
&p_1 y_1 + p_2 y_2 - \\
&\left[
\begin{aligned}
&Max\left((p_1 - c)K_b, 0\right) + \\
&Max\left((p_2 - c)K_b, 0\right) + \\
&Max\left((p_1 + p_2)K_a - \mathrm{F}_a, 0\right)
\end{aligned}
\right]
\end{aligned}
\right)
$$

A related version with different assumptions about the relation of costs illustrates the different solutions that can arise in the conditional dual and conditional market-clearing pricing problems in the second period.

Two-Period Sequential Update

Plant a has only fixed costs,
b has only variable costs.



2nd Period Conditional
Price Problem

$$v_p^\tau\left(y \middle| p_1^*\right) = \underset{p_2}{Max}\begin{pmatrix} p_1^* y_1 + p_2 y_2' - \\ \begin{bmatrix} Max\left(\left(p_1^* - c\right) K_b, 0\right) + \\ Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(\left(p_1^* + p_2\right) K_a - \mathrm{F}_a, 0\right) \end{bmatrix} \end{pmatrix}$$

2nd Period Conditional
Market Clearing Problem

$$v_z^\tau\left(y \middle| \left(p_1^*, x_{a1}^*, x_{b1}^*, u_{a1}^*\right) = \left(c, 0, K_b, 0\right)\right)$$

$$= \underset{p_2}{Max}\begin{pmatrix} p_1^* y_1 + p_2 y_2' - \\ \begin{bmatrix} Max\left(\left(p_2 - c\right) K_b, 0\right) + \\ Max\left(p_2 K_a - \mathrm{F}_a, 0\right) \end{bmatrix} \end{pmatrix}$$

References

Bacci, T., Frangioni, A., Gentile, C., & Tavlaridis-Gyparakis, K. (2019). New MINLP Formulations for the Unit Commitment Problems with Ramping Constraints. Retrieved from http://www.optimization-online.org/DB_FILE/2019/10/7426.pdf

Biggar, D., & Hesamzadeh, M. (2020). On Multi-Interval Real-Time Markets, (January), 1–27.

Cadwalader, M. D., Gribik, P. R., Hogan, W. W., & Pope, S. L. (2010). Extended LMP and Financial Transmission Rights. Retrieved from http://www.hks.harvard.edu/fs/whogan/CHP_ELMP_FTR_060910.pdf

Chao, H. (2019). Incentives for efficient pricing mechanism in markets with non-convexities. *Journal of Regulatory Economics*. https://doi.org/10.1007/s11149-019-09385-w

Gribik, P. R., Hogan, W. W., & Pope, S. L. (2007). Market-Clearing Electricity Prices and Energy Uplift. Retrieved from http://www.hks.harvard.edu/fs/whogan/Gribik_Hogan_Pope_Price_Uplift_123107.pdf

Hogan, W. W. (2016). *Electrcity Market Design: Optimization and Market Equilibrium*. Retrieved from https://www.hks.harvard.edu/fs/whogan/Hogan_UCLA_011316.pdf

Hogan, W. W., & Pope, S. L. (2017). Priorities for the Evolution of an Energy-Only Electricity Market Design in ERCOT. Retrieved from https://scholar.harvard.edu/whogan/files/hogan_pope_ercot_050917.pdf

Hua, B., & Baldick, R. (2017). A Convex Primal Formulation for Convex Hull Pricing. *IEEE Transactions on Power Systems*, *32*(5), 3814–3823. https://doi.org/10.1109/TPWRS.2016.2637718

Hua, B., Schiro, D. A., Zheng, T., Baldick, R., & Litvinov, E. (2019). Pricing in Multi-Interval Real-Time Markets. *IEEE Transactions on Power Systems*, *34*(4), 2696–2705. https://doi.org/10.1109/TPWRS.2019.2891541

Knueven, B., Ostrowski, J., Castillo, A., & Watson, J. (2019). A Computationally Efficient Algorithm for Computing Convex Hull Prices. *Optimization OnLine*, 1–19. Retrieved from http://www.optimization-online.org/DB_HTML/2019/08/7327.html

Knueven, B., Ostrowski, J., & Wang, J. (2018). The Ramping Polytope and Cut Generation for the Unit Commitment Problem. *INFORMS Journal on Computing*, *30*(4), 739–749. https://doi.org/10.1287/ijoc.2017.0802

MISO. (2019). ELMP III White Paper I: R&D report and Design Recommendation on Short-Term Enhancements. MISO. Retrieved from https://cdn.misoenergy.org/20190117 MSC Item 05 ELMP III Whitepaper315878.pdf

Schiro, D. A. (2017). Flexibility Procurement and Reimbursement: A Multi-period Pricing Approach. *FERC Technical Conference*. Retrieved from https://www.ferc.gov/CalendarFiles/20170623123635-Schiro_FERC2017_Final.pdf

Schweppe, F. C., Caramanis, M. C., Tabors, R. D., & Bohn, R. E. (1988). *Spot pricing of electricity*. Kluwer Academic Publishers. Retrieved from http://books.google.com/books?id=Sg5zRPWrZ_gC&pg=PA265&lpg=PA265&dq=spot+pricing+of+electricity+schweppe&source=bl&ots=1MIUfKBjBk&sig=FXe_GSyf_V_fcIuTmUtH7mKO_PM&hl=en&ei=Ovg7Tt66DO2x0AH50aGNCg&sa=X&oi=book_result&ct=result&resnum=3&ved=0CDYQ6AEwAg#v=onep

Yu, Y., Guan, Y., & Chen, Y. (2019). An Integral Formulation and Convex Hull Pricing for Unit Commitment. Retrieved from http://arxiv.org/abs/1906.07862

Zhao, J., Zheng, T., & Litvinov, E. (2019). A Multi-Period Market Design for Markets with Intertemporal Constraints, 1–11. Retrieved from http://arxiv.org/abs/1812.07034

Zheng, T., Zhao, F., Schiro, D., & Litvinov, E. (2018). The Hidden Properties of Fast Start Pricing. *FERC Increasing Market and Planning Efficiency and Enhancing Resilience through Improved Software*. Retrieved from https://www.ferc.gov/CalendarFiles/20180626080613-T1 - 3 - HiddenPropertiesOfFSP-TZheng_ISONE.pdf

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