

# Investigation of Convex Hull Pricing at Midwest ISO

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# MISO's Current Pricing Method

- Midwest ISO market structure:
  - Bid-based day-ahead market using security constrained unit commitment and economic dispatch (SCUC and SCED);
  - Bid-based reliability commitment process using SCUC to ensure that sufficient capacity will be on-line to meet forecast real-time requirements;
  - Bid-based real-time security constrained economic dispatch.
- Locational prices for day-ahead and real-time settlements determined in SCED with fixed commitment as set in SCUC.
  - The price at a location is the marginal cost of meeting the energy requirement (or ancillary service requirement) in the SCED.
  - Prices are produced by solving the dual of the SCED problem.

# Basis for Current Locational Prices

- Assume fixed commitment.
  - Microeconomics:
    - The prices are market clearing prices.
      - Each generator (load) would want to produce (consume) as scheduled to maximize its profits.
    - The prices are efficient prices.
      - All generators and loads would want to follow the schedule, which maximizes societal surplus.
  - Game theory (market games):
    - Using the prices to allocate the societal surplus to generators, loads, and transmission rights holders eliminates incentives to leave the pool dispatch.
      - No subset of generators and loads could improve its overall position by scheduling outside the market.
      - The allocation of societal surplus using the prices is in the “core” of the market game.

# Effects of Adding Commitment to the Mix

- Make commitment decisions part of the problem.
  - Microeconomics:
    - There usually are not market clearing prices.
      - Cannot set prices which would incentivize profit maximizing generators (loads) to produce (consume) at scheduled levels.
      - Under MISO's current pricing method, start-up costs and no-load costs do not affect the market prices.
        - » MISO pays uplifts to ensure that generators are not paid less than their bid costs for production.
        - » This is RSG in MISO.
    - Game theory (market games):
      - There may not be prices that can be used to allocate societal surplus that completely eliminate any incentive for parties to self schedule.
  - MISO worked with LECG to develop a pricing approach to address these and other shortcomings.

# Convex Hull Pricing

- Convex hull prices (CHPs) are calculated by solving the dual of the SCUC problem rather than the SCED problem.
- Convex hull prices:
  - Minimize the additional payments (uplifts) needed to incentivize profit maximizing generators (loads) to produce (consume) at scheduled levels.
  - Incorporate start-up costs and no-load costs in the prices.
  - Minimize incentives for parties to leave the pool commitment and dispatch to self schedule when convex hull prices are used to allocate societal surplus.
    - Any additional profit that a coalition could achieve by self-scheduling outside the market is bounded by the uplifts described above.
    - Since finding trading partners with whom to self-schedule is not without costs, this tends to reduce any incentive to schedule outside the market.

# SCUC

- We can write a stylized version of SCUC:

$$\min_{\mathbf{g}, \mathbf{d}} \sum_t \left[ \sum_i GenCost_{it}(g_{it}) - \sum_j LoadVal_{jt}(d_{jt}) \right]$$

subject to

$$\mathbf{e}^T \mathbf{g}_t - \mathbf{e}^T \mathbf{d}_t - \mathbf{LossSen}_t^T (\mathbf{d}_t - \mathbf{g}_t) = -Offset_t \quad \forall t$$

$$\nabla Flow_{kt}^T (\mathbf{g}_t - \mathbf{d}_t) \leq F_{kt}^{max} \quad \forall k, t$$

Constraints on individual generators  $i$

Constraints on individual demands  $j$

# Dual for SCUC (CHP Problem)

- We can form a dual for the SCUC:

$$\left. \begin{array}{l}
 \max_{\lambda, \mu} \left\{ \sum_t \lambda_t (-Offset_t) + \sum_t \sum_k \mu_{kt} F_{kt}^{max} \right. \\
 \left. + \left\{ \begin{array}{l}
 \min_{g, d} \left[ \sum_t \sum_i GenCost_{it}(g_{it}) - \sum_t \sum_j LoadVal_{jt}(d_{jt}) \right. \right. \\
 \left. \left. - \sum_t \lambda_t (\mathbf{e}^T \mathbf{g}_t - \mathbf{e}^T \mathbf{d}_t - \mathbf{LossSen}_t^T (\mathbf{d}_t - \mathbf{g}_t)) - \sum_t \sum_k \mu_{kt} \nabla Flow_{kt}^T (\mathbf{g}_t - \mathbf{d}_t) \right] \right. \\
 \left. \begin{array}{l}
 \text{subject to} \\
 \text{Constraints on individual generators } i \\
 \text{Constraints on individual demands } j
 \end{array}
 \right\}
 \end{array} \right\}$$

subject to

$$\mu_{kt} \leq 0 \quad \forall t$$

- The convex hull prices are  $\lambda_t (\mathbf{e} + \mathbf{LossSen}_t) + \sum_k \mu_{kt} \nabla Flow_{kt}$

# Solving for Convex Hull Prices

- The SCUC dual is a difficult problem:
  - Its objective function is non-differentiable;
  - It is very large scale for the MISO's system:
    - Around 5000 transmission constraints;
    - Around 1000 generators;
    - 24 coupled hours in day-ahead.
- MISO worked with consultants to test several classes of solution techniques:
  - Subgradient descent methods;
  - Cutting plane methods.
    - Analytic Center Cutting Plane appears to be most promising.



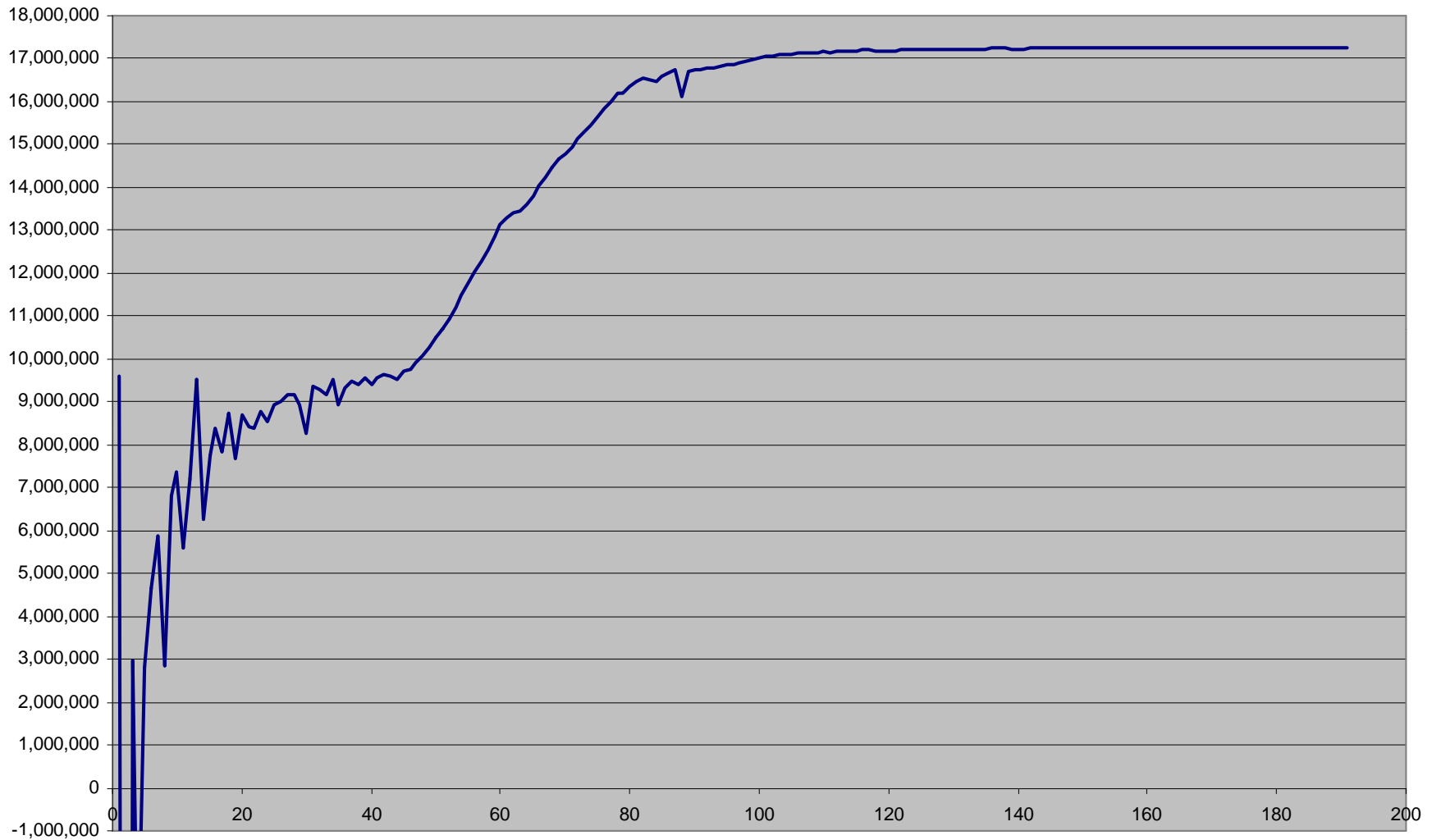
# Handling Transmission Constraints

- MISO's system has around 5000 transmission constraints, however very few are binding in SCUC/SCED in any hour.
  - Usually less than 10 transmission constraints are binding in a given hour.
- To reduce the number of transmission constraints in the CHP problem, we selected binding and near binding transmission constraints in the SCUC/SCED solution.

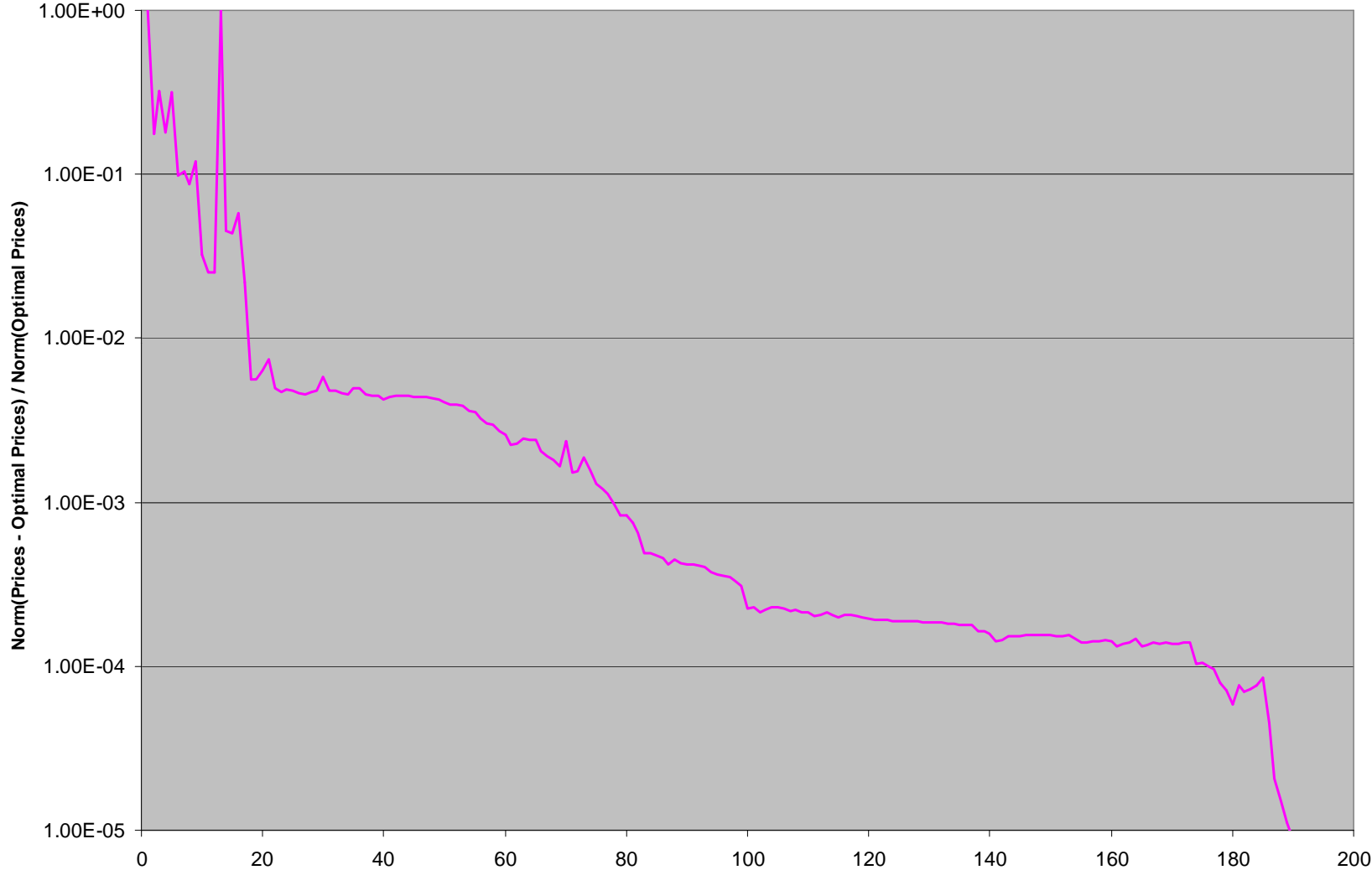
# Example Problem

- Example problem from November 2007.
  - Day-Ahead market (energy only).
  - Number of generators: 1009
  - Number of transmission constraints modeled: 18
  - Study period: 24 hours.
- Computer used: 64 bit desk top, 2.33GHz, 4GB of RAM running Windows XP 64 bit.
- Performance summary:
  - Number of iterations: 191
  - Solution time: 223 seconds
- The software was for proof of concept. It was not optimized for speed.
  - LPs solved using MATLAB. No performance enhancement methods such as hot-starting the LPs using solution from previous iteration were used.
  - All indications are that problem can be solved in practice.

# Dual Objective Function Value by Iteration



# Convergence of Dual Price Vector



# Comparison of Convex Hull Prices and Current Prices

- Convex hull prices at reference node compared to current prices at reference node:

Hour	1	2	3	4	5	6	7	8	9	10	11	12
Current Price	17.9	16.6	16.7	16.7	16.8	16.2	16.4	20.5	20.4	21.2	22.4	21.1
CHP	20.6	19.1	18.5	18.1	18.8	18.5	19.6	20.5	22.9	23.8	24.6	24.3
Difference	2.7	2.5	1.8	1.4	2.0	2.3	3.2	0.0	2.5	2.6	2.2	3.2

Hour	13	14	15	16	17	18	19	20	21	22	23	24	Average
Current Price	20.8	20.5	20.4	20.6	20.5	23.2	48.3	40.0	26.3	21.3	21.1	18.4	21.8
CHP	23.1	22.2	20.5	20.2	21.0	25.1	57.0	35.9	29.0	25.5	24.2	23.2	24.0
Difference	2.3	1.7	0.1	-0.4	0.5	1.9	8.7	-4.1	2.7	4.2	3.1	4.8	2.2

- The average convex hull price at the reference node is \$24.0/MWh compared to the average reference node price of \$21.8/MWh using current pricing structure.
  - The increase of \$2.2/MWh reflects the effects of including start-up and no-load costs in setting prices.

# Why Include Near-Binding Constraints

- MISO may have to commit a generator to manage a transmission constraint.
  - If the generator is not committed, the flow may exceed the transmission limit.
- With the generator committed, the flow may be under the limit.
  - Current pricing method would set a shadow price of zero on the constraint since it is not binding in SCED.
    - No price difference exists between nodes on either side of the constraint even though costs are incurred to manage the constraint.
  - Convex hull pricing sets a non-zero shadow price on the constraint based on the need to commit a unit to manage the constraint.
    - The constraint is “financially binding” and price differences exist between nodes on different sides of the constraint.
    - This has been observed in example problems.

# Current Work

- MISO is working with LECG to address integrating Day-Ahead and Real-Time markets.
  - Handling of uplifts arising in the two markets to ensure that a participant is not paid twice for the same action.
  - Determining which resources and costs to include when calculating Real-Time CHPs.
- MISO is also working with software vendors.
  - Complete the incorporation of ancillary services in the CHP software.
  - Improve solution algorithms.

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